

The Three Rs of Experimentation:*

Recording the Details, Reducing the Data, Reporting the Results

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Recording The Details In A Permanent Notebook

1.1 Why is the laboratory notebook important?

An essential part of the process of learning to do experimental physics is the development of your skill at *completely* and *truthfully* documenting an experiment, so that you will be able to share the results of your work *convincingly* with the scientific community. Although the laboratory notebook is primarily for your own use, as a record of what you have done, it is worth reminding ourselves that experimental physics is a communal endeavor, an important purpose of which is to add to our body of knowledge about how the physical universe works. Whether you work in an industrial laboratory, or in an academic setting, the outcome of your work is usually reported in some form, either written or oral, to your peers or to those who funded your work. Thus, recording all the details of your experiment in an orderly fashion, as the work unfolds, will make it easier for you to recall the essential details when it comes time to report the results. If you make an important discovery, a well-kept lab notebook may also serve as a convincing legal document, which could be useful in obtaining patent agreements, Nobel prizes, etc.

1.2 What are the elements of a good laboratory notebook?

To produce a complete and convincing document of your experiment requires first that you keep records in the laboratory notebook as the experiment progresses. A good lab notebook should be like a journal or diary, in so far as it contains not just tables of data but your thoughts on what you are doing and why you are doing it. It is absolutely essential that you record all of your data in the notebook *as the measurements are made*. You should also be liberal in the use of written comments throughout the notebook which will help you (and the grader!) to understand what you have done.

A second essential element of a complete notebook is that it should contain all details of data analysis, including any graphs of data and calculations of experimental errors. The notebook should show clearly how you got from the raw measurements to the final results. All important equations should be written out, and a sample calculation of each type should be shown. Graphs may be produced by computer, but must be permanently attached into the notebook (with staples, tape, or glue) and clearly cross-referenced to the appropriate data tables, so that the reader may understand what data are being plotted.

Since all measurements have some degree of uncertainty, and all experimental results are derived from measurements, it follows that all experimental results will have some degree of uncertainty. As part of the data analysis, your notebook must also show clearly how you determined the uncertainty in the final results. This important process is discussed in more detail in Chapter 2.

Finally, you should get into the habit of collecting your thoughts at the end of each lab session by writing a brief summary of what you have accomplished and what you plan to do next. Just as a daily review of lecture notes makes it easier to study for an exam, the daily summary of your lab work should help you stay on top of the analysis and will make it easier to prepare a report. Whether or not a formal report is required, you should devote the last page or so in your notebook to a discussion of the important results of the experiment. Tell what the results mean and what conclusions can be drawn from them.

1.3 Some ground rules for record keeping

a. Produce a truthful record of your experiment

- **Honesty has its benefits:** First and foremost your lab notebook must be an honest reflection of the work that you did, recorded at the time that you actually did the work. All students in this course are required to be present in the lab for all of the experiments, and to record their own data. There is a very pragmatic aspect to truthfulness in record keeping. It helps to verify your claim to a particular discovery and ensure such things as patent rights. Standard procedure in industry is for you to sign your notebook entries and have them countersigned and dated by a co-worker on a daily basis. Thus the laboratory notebook may function as a legal document, as well as being a record of your work for your own information.
- **Dishonesty is not tolerated:** The scientific community demands that its members do their work with integrity. Under no circumstances will any student be allowed to copy data from another student without actually doing the experiments. If you have any doubts about the serious problems that can arise when lab notes are allegedly not kept with integrity, see the story which appeared on the front page on the *New York Times* on December 3rd, 1991, and related articles which appeared in the 'Science Times' section of the *New York Times* on May 15, 1990 and June 4, 1992 regarding allegations of fraud which led to the resignation of a university president.

b. Produce a permanent record of your experiment

- **Use bound notebooks:** In this and other physics lab courses at Carnegie Mellon you are required to keep records of your experiments in bound notebooks. It is not acceptable to keep records in spiral or loose leaf notebooks, from which it is tempting to tear a page out if you make a mistake.
- **Write in ink; record everything; erase nothing:** The lab notebook should be a permanent record of everything you did in the lab. If you make a mistake, just cross it out lightly and note the reason for doing so. Never erase, obliterate, or tear out anything from the notebook. You should develop this habit for two reasons: first, to ensure verifiable intellectual integrity (e.g. you can't claim to have made a new discovery simply by omitting half of your data) and second, you may find that you were right in the first place and want to retrieve the data.
- **No loose sheets of paper:** It is not acceptable to keep records on loose sheets of paper, which may be lost or misplaced easily. The instructors of this course reserve the right to confiscate any loose sheets of paper that the student may be using instead of a proper lab notebook. Graphs, computer printouts, and other important information relevant to the experiment should be attached permanently into the notebook using staples, tape, or glue, and not simply stuffed between the pages.

c. Produce a complete and convincing record of your experiment

- **Write legibly and with sufficient detail:** In order for a piece of research to be convincing, it must be understandable and reproducible. To that end, your notebook must contain enough information, written with sufficient clarity, for someone who is familiar with your field of work to be able to follow your notes and understand (and if necessary, reproduce) what you did. Thus, the notebook must be legible, but it is not expected to be perfectly neat and free of (corrected) mistakes.
- **Be concise in discussion of the theory:** In order to be a complete record of your experiment, it is not necessary for the notebook to include every word that was ever written on the underlying theory. Be concise. Include just enough theory to motivate your experiment. If you feel that additional information is needed, you can include a reference to a primary source.

- **Keep your own independent notes:** Each individual must keep his or her own notebook for each experiment, and to write all of the original data in his/her notebook by hand. We want to encourage cooperation and teamwork in the laboratory, but we do not want to encourage you to become completely dependent on your lab partner to have written down all the important information that you did not record. It is not acceptable for one partner to be the record-keeper for the team. Nor is it acceptable for the other partner to put a computer print-out of the data in his/her notebook and to write “see partner’s notebook for the original data.” This applies not only to original data, but also to analysis, graphs, error calculations, summaries, and conclusions. Do all of your own work in your own notebook.
- **Be sure that you have taken enough data:** In order to be convincing, the conclusions which you reach must be supported by the data. To be sure that you have enough data to support your conclusions, you must analyze the data as you go along. Don’t leave the lab until you have plotted the data that you took that day. Don’t wait until the day before the assignment is due to begin your analysis, or it may be too late to correct mistakes.

1.4 Notebook guidelines

Presented below is a list of specific guidelines on how to document your experiments. The guidelines serve a dual purpose. First, they outline some of the criteria which will be used in evaluating your performance in this course. Second, and more important, these guidelines will help you to develop good laboratory habits, which can save you time and ensure that you have an accurate and useful record of your work.

In order to help these guidelines ‘come to life’ we include an example of a laboratory notebook in Section 1.5. The important points are summarized in the Laboratory Notebook Checklist, Section 1.6.

- **Title page:** The front cover of your notebook should show your name, the title of the experiment and the name of your lab partner. (See example notebook cover page)
- **Index:** An index can be very useful when it comes time to prepare some sort of formal communication based on your work (e.g. a publication, poster, seminar, etc). It can save you a lot of time when you want to find key results, particularly if the research extends over a long period of time and fills many pages of a notebook.
- **Purpose:** State clearly (but briefly) at the beginning what the experiment is about and what you expect to achieve. This might be phrased in the form of a hypothesis (e.g. “We expect the period of a simple pendulum to vary as the square root of the length.”) or as a specific goal (e.g. “to measure the speed of light”).

- **Theory:** The important physics underlying the experiment must be shown in equation form, with all of the symbols defined. You do not need to include a complete derivation of the theory. In most cases it is sufficient to show the key equation which motivates the experiment.
- **Method:** Include a brief explanation of what kind of measurements you will make and what sort of analysis will be done to obtain the final results. It may be a matter of style, whether you want to describe the method separately, or combine it with the diagram of the apparatus.
- **Diagram of apparatus:** Show enough detail to make it clear what equipment will be used and what measurements will be made. It's also a good idea to include manufacturer and model numbers of commercially made equipment. The diagram can be very schematic in nature. Use block diagrams with names and model numbers. It is not necessary to spend time creating a photo-realistic rendering of what each piece of equipment looks like.
- **Dated log entries:** Record the date of each day's work (and possibly also the time). It's also good practice to focus your thoughts and write a brief outline of the plan for the day.
- **Data:** Record all of the original data. Make use of table headings, which include the name or symbol of the measured quantity, the units of measurement, and estimates of uncertainty.
- **Written comments:** Be liberal in writing down your thoughts and qualitative observations as you go along. Written comments can be just as valuable as numbers when it comes to interpreting the final results.
- **Data analysis (equations):** Every physics equation that you use to analyze your data must be written in your notes, with each symbol defined.
- **Data analysis (graphs):** Graphs must show the data points with error bars and (usually) a computer-generated fit to the data. The axes of the graph must be labeled with the physical quantity and the units. The title of the graph should be descriptive of the purpose of the plot, and not something insultingly obvious, such as "y vs. x"
- **Error analysis:** The equations used for error analysis must also be written in your notes, with symbols appropriate to the specific situation. (You do not need to write the equations for mean and standard deviation.)
- **Final results: summary table and significant digits:** The final results should be clearly distinguished from the raw data in your lab notes. The use of summary tables, or some creative way of highlighting the key results will help you immensely when it comes time to write a report. It is common practice to write a result as the value plus

or minus its uncertainty ($x \pm \sigma_x$). It is also helpful to record the final result in such a way as to make it immediately obvious how closely your measurement agrees with the expected value. Thus, a table such as that in table 1.1 (below), which includes the expected value, would be most appropriate.

Experimental result for g	$9.808 \pm 0.040 \text{ m/s}^2$
Published value of g for Pittsburgh, PA	9.80118 m/s^2
Comparison (Δ/σ)	0.17 (good agreement)

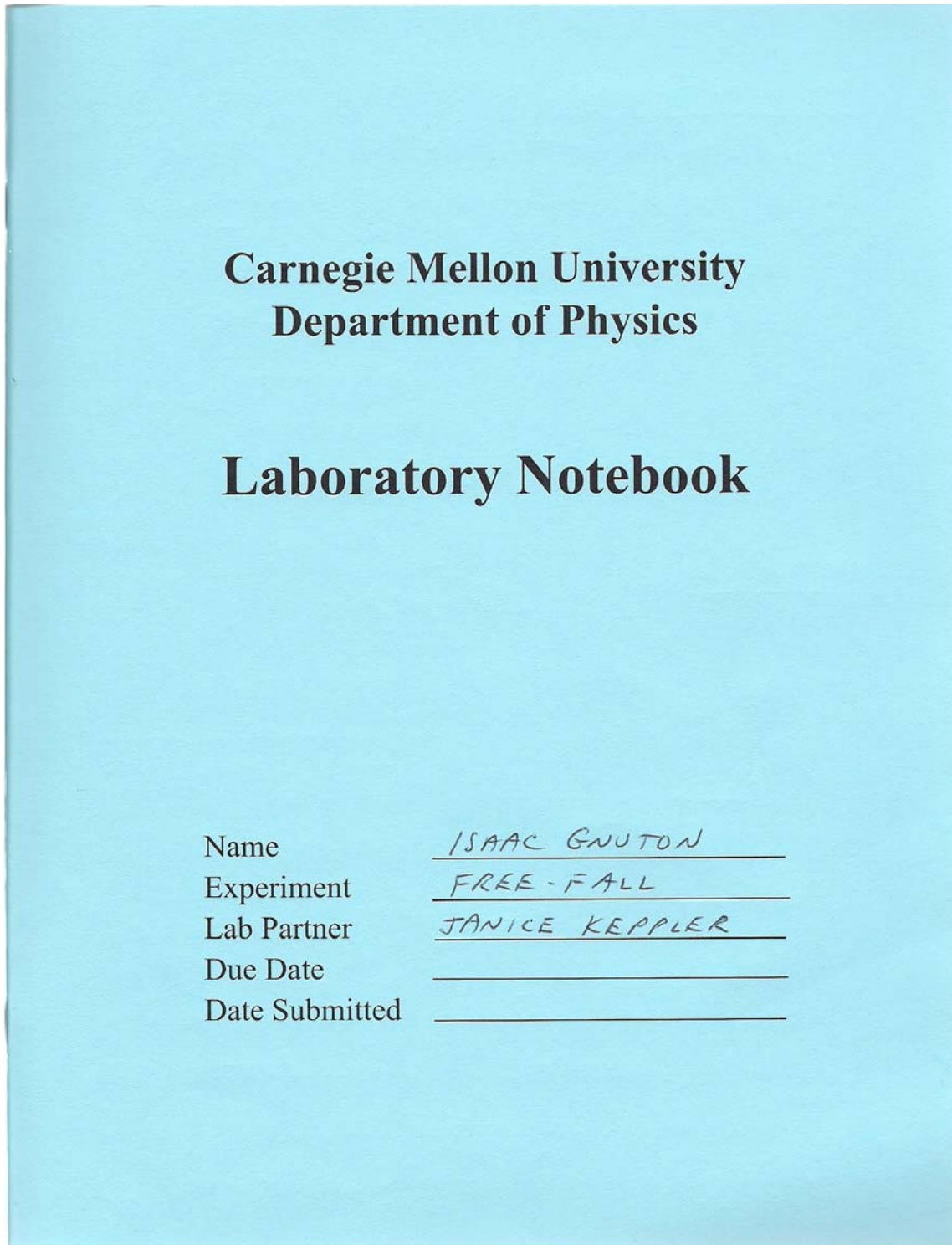
Table 1.1 Final results for the free-fall experiment (see the sample laboratory notebook). The published value of g is taken from Hugh D. Young, University Physics, 8th ed., p.336. The significance of the expression Δ/σ will be discussed in Chapter 2, on Treatment of Experimental Errors.

Note that the experimental result in Table 1.1 conforms to the convention, which we will adopt in this course, and which is common in professional publications, in regard to the number of significant digits to report. The uncertainty should be rounded off to two significant digits. The result itself should be written in the same format as the uncertainty (i.e. same power of ten, if using scientific notation, and same number of decimal places). The uncertainty in the published value of g is at least two orders of magnitude smaller than the experimental uncertainty, as implied by the number of significant digits.

- **Discussion and conclusions:** Write a brief discussion of what you accomplished at the end of your notes. Focus on the key physics of the experiment. Restate what you set out to accomplish. Did your results support the hypothesis (or agree with a previously published result)? If you made any simplifying assumptions, do your results indicate that these were valid assumptions? If the results did not support the hypothesis (or otherwise disagreed with expectations), think carefully and offer a plausible explanation for the observed difference.

1.5 Example Laboratory Notebook

Cover Page: Include your name, a descriptive title of the experiment, and your partner's name.



The image shows a light blue notebook cover. In the center, the text "Carnegie Mellon University" and "Department of Physics" is printed in a bold, black, serif font. Below this, the title "Laboratory Notebook" is printed in a larger, bold, black, serif font. At the bottom of the cover, there is a form with five rows, each consisting of a label on the left and a horizontal line on the right. The labels are "Name", "Experiment", "Lab Partner", "Due Date", and "Date Submitted". The first three rows have handwritten text in black ink: "ISAAC GNUTON" for Name, "FREE-FALL" for Experiment, and "JANICE KEPPLER" for Lab Partner. The last two rows are empty.

Name	<u>ISAAC GNUTON</u>
Experiment	<u>FREE-FALL</u>
Lab Partner	<u>JANICE KEPPLER</u>
Due Date	<u> </u>
Date Submitted	<u> </u>

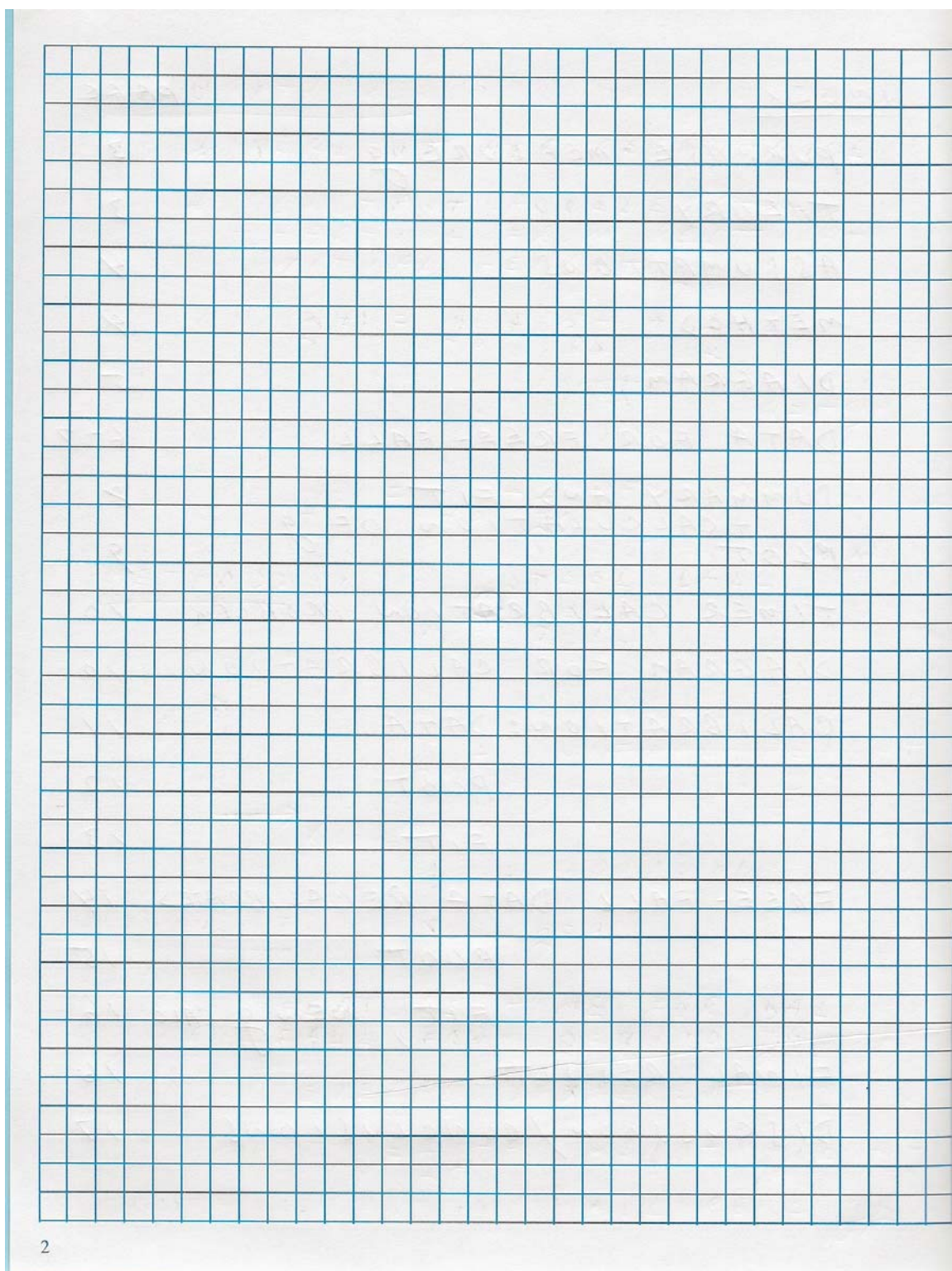
Example Laboratory Notebook Page 1: Index

An index page is not required, and will not be a grading criterion for this course. However, keeping an index is highly recommended. An index page can be a great time-saver, when it comes to retrieving important information from a long experiment.

<u>INDEX</u>	<u>PAGE</u>
PURPOSE - MEASURE g	3
THEORY	3
ASSUMPTIONS	4
METHOD	4
DIAGRAM	5
DATA FOR FREE-FALL	6-7
SUMMARY AND FIT + CALCULATION OF g	8
PLOT	9
TIMER CALIBRATION PROBLEM	10
DIAGRAM FOR CALIBRATION	10
CALIBRATION: DATA	11
PLOT	12
FIT	13
FREE-FALL DATA RECALIBRATED	14
PLOT	15
FIT, NEW g CALC	16
FINAL RESULT	16
DISCUSSION / CONCLUSIONS	17

Example Laboratory Notebook Page 2 (blank)

Page 2 would have been used to continue the index, if the experiment had been longer. (Again, this is a matter of personal style, and not a grading criterion for the course.)



Example Laboratory Notebook Page 3

Include a brief statement of the purpose or goal of the experiment, and just enough of the theory to motivate the measurements that will be done.

5 MAY 2010

PURPOSE: TO DETERMINE THE LOCAL ACCELERATION DUE TO GRAVITY, g

THEORY:

- UNIVERSAL LAW OF GRAVITATION PREDICTS FORCE BETWEEN TWO MASSES, m_1 AND m_2
$$F_{\text{grav}} = \frac{G m_1 m_2}{r^2}$$

(r = DISTANCE
 G = GRAVITATIONAL CONSTANT)
- OBJECT OF MASS m FALLING NEAR SURFACE OF EARTH (MASS M) ACCELERATES ACCORDING TO NEWTON'S 2ND LAW:
$$F = ma = -mg$$
$$g = \frac{GM}{r^2}$$

r = RADIUS OF EARTH
- WHY MEASURE g LOCALLY?
 - * EARTH IS A ROTATING REFERENCE FRAME $\Rightarrow g$ DEPENDS ON LATITUDE
 - * EARTH DOES NOT HAVE UNIFORM DENSITY OR ALTITUDE $\Rightarrow g$ DEPENDS ON LOCATION
- PUBLISHED VALUE: $g = 9.80118 \text{ m/s}^2$
(H.D. YOUNG, UNIVERSITY PHYSICS 8th ed. p.336)

3

Example Laboratory Notebook Page 4

Include a statement of key assumptions, initial conditions, etc. Also give a brief overview of how the measurements will be made, and what analysis will be done with the data.

ASSUMPTIONS:

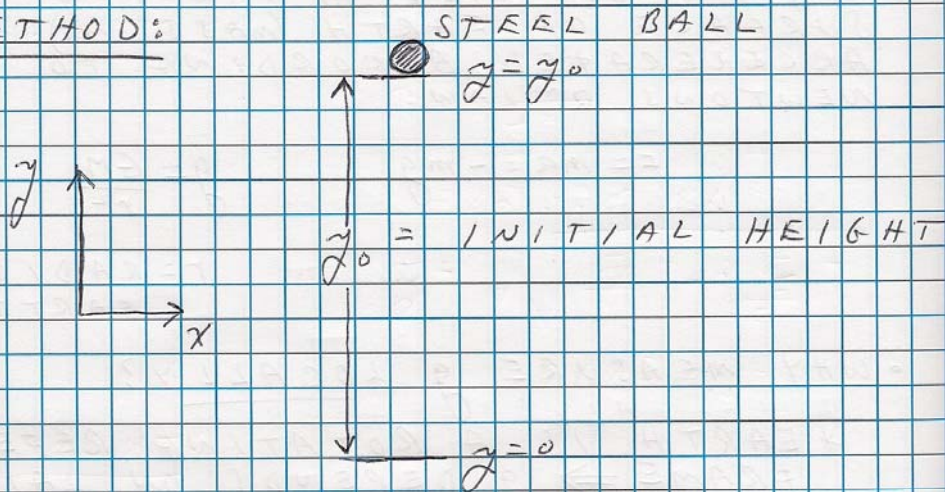
- * NEGLECT AIR RESISTANCE
- * CONSTANT ACCELERATION STARTING FROM REST, AT INITIAL HEIGHT y_0

$$y(t) = y_0 + v_0 t + \frac{1}{2} a t^2$$

$$a = -g$$

$$y(t) = y_0 - \frac{1}{2} g t^2$$

METHOD:



MEASURE TIME FOR BALL TO FALL FROM $y = y_0$ TO $y = 0$ FOR

SEVERAL DIFFERENT y_0 :

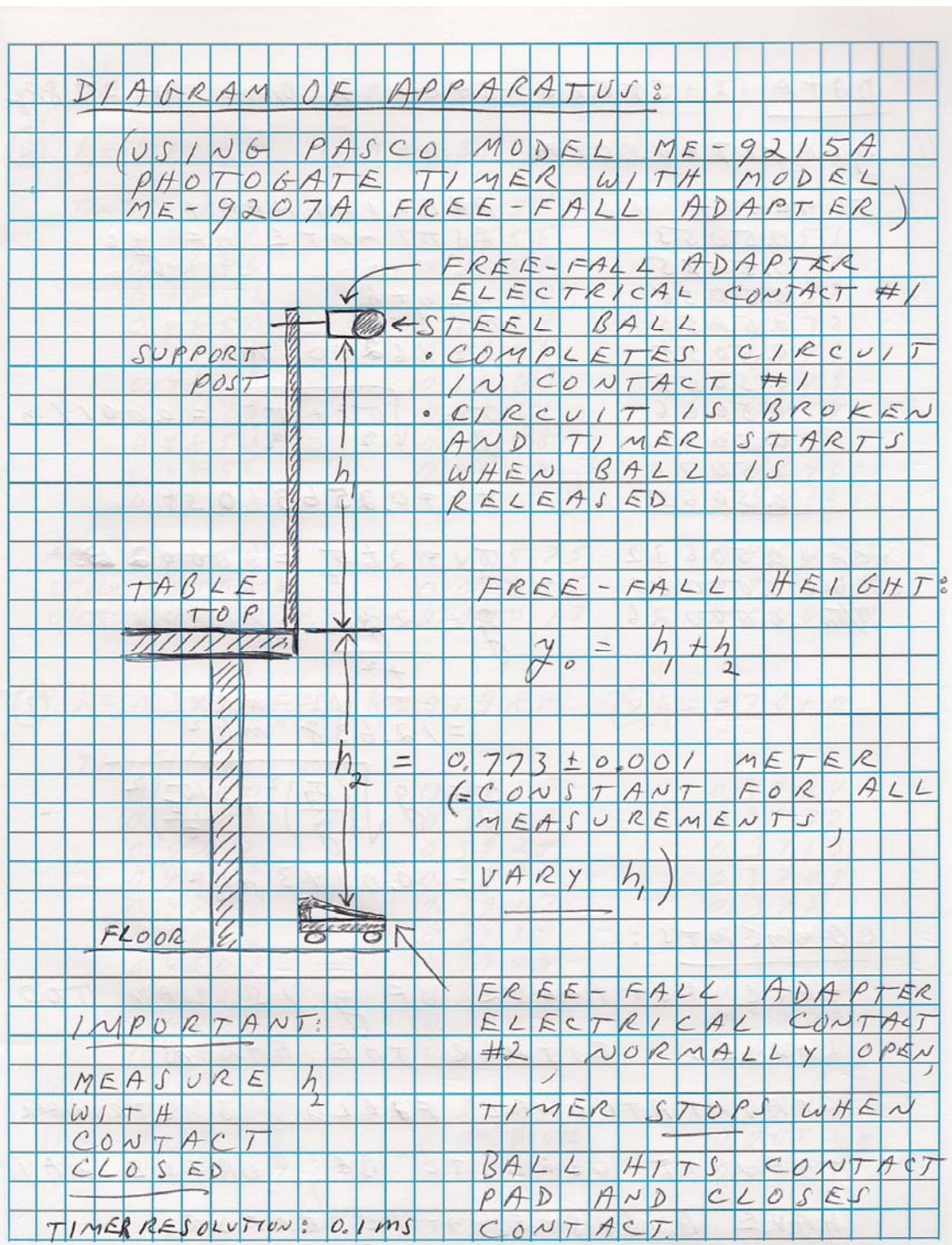
$$y_0 = \frac{1}{2} g t^2$$

• PLOT y_0 vs t^2

• SLOPE = $g/2$

Example Laboratory Notebook Page 5

A diagram of the apparatus must be included for each part of the experiment. It need not be a photographically accurate representation of all of the equipment. A schematic, with enough information to understand the key components and how the measurements are made, is all that is necessary.



Example Laboratory Notebook Page 6

Data must include the name or symbol of the physical quantity, as well as units and uncertainty. The key equations used to analyze the data and the uncertainties must be shown algebraically. Be liberal in the use of written comments, as well as numbers.

<u>DATA</u>	STEEL BALL $\frac{3}{4}$ " DIAM. $m = 27.88g$
① $h_1 = 0.847 \pm 0.001 m$	
<u>TIMES (s)</u>	<u>PRELIMINARY ESTIMATE OF g:</u>
0.5053	
0.5075	$y = h_1 + h_2$
0.5058	$= 1.6200 m$
0.5055	
0.5058	$\sigma = \sqrt{\sigma_{h_1}^2 + \sigma_{h_2}^2} = 0.0014 m$
0.5057	
0.5066	$t^2 = 0.256360 s^2$
0.5074	
0.5071	$\sigma_{t^2} = 2t\sigma_t = 0.00026 s^2$
0.5065	
MEAN 0.50632	$g = \frac{2y_0}{t^2}$
ST. DEV. 0.00081	$= 12.638 m/s^2$
σ/\bar{y}_0 0.00026	$\sigma_g = g \sqrt{\left(\frac{\sigma_y}{y}\right)^2 + 2\left(\frac{\sigma_{t^2}}{t^2}\right)^2}$
	$= 0.0143 m/s^2$
<u>COMMENTS:</u>	
THIS ESTIMATE OF g IS WAY TOO LARGE. EITHER THE EARTH'S GRAVITATIONAL FIELD IS STRONGER THAN IT USED TO BE, OR WE MAY HAVE A SYSTEMATIC ERROR.	

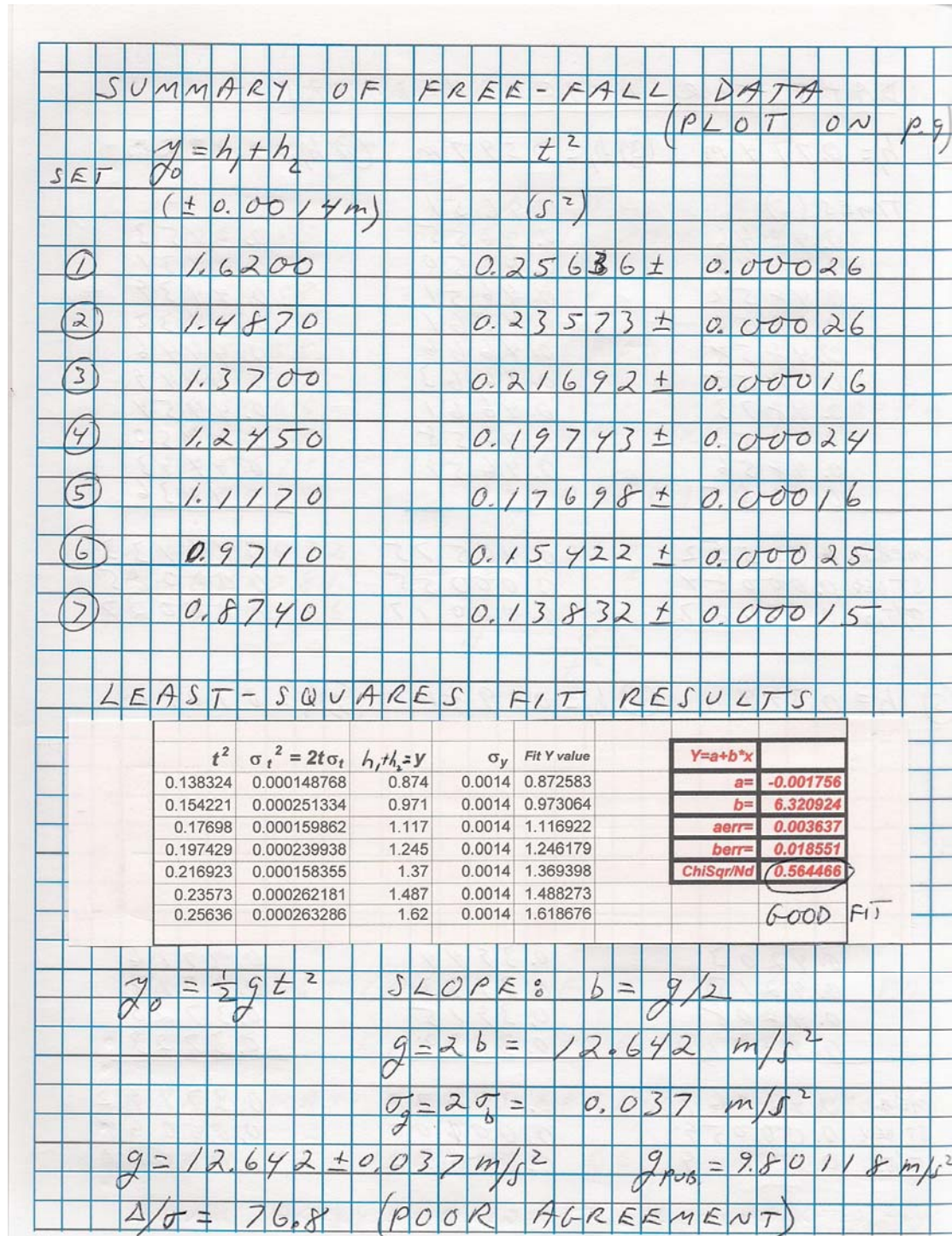
Example Laboratory Notebook Page 7

More data of the same type as on Page 6. The uncertainty on the heights is assumed to be the same as the uncertainty on the height for data set 1, and need not be repeated here.

DATA FOR OTHER HEIGHTS:									
(2) $h_1 = 0.714 \text{ m}$			(3) $h_1 = 0.597 \text{ m}$			(4) $h_1 = 0.472 \text{ m}$			
TIMES (s)									
0.4846			0.4651			0.4453			
0.4846			0.4655			0.4431			
0.4852			0.4650			0.4439			
0.4850			0.4651			0.4432			
0.4858			0.4661			0.4446			
0.4853			0.4666			0.4449			
0.4873			0.4663			0.4454			
0.4853			0.4661			0.4450			
0.4856			0.4658			0.4443			
0.4865			0.4659			0.4436			
MEAN 0.48552			0.46575			0.44433			
ST. DEV. 0.00084			0.00055			0.00085			
$\sigma/\sqrt{10}$ 0.00027			0.00017			0.00027			
(5) $h_1 = 0.344 \text{ m}$			(6) $h_1 = 0.198 \text{ m}$			(7) $h_1 = 0.101 \text{ m}$			
TIMES (s)									
0.4200			0.3917			0.3724			
0.4200			0.3919			0.3732			
0.4211			0.3924			0.3713			
0.4208			0.3936			0.3709			
0.4207			0.3917			0.3721			
0.4216			0.3934			0.3716			
0.4203			0.3944			0.3716			
0.4215			0.3944			0.3718			
0.4208			0.3933			0.3720			
0.4201			0.3915			0.3723			
MEAN 0.42069			0.39271			0.37192			
ST. DEV. 0.00059			0.00040			0.00064			
$\sigma/\sqrt{10}$ 0.00019			0.00032			0.00020			

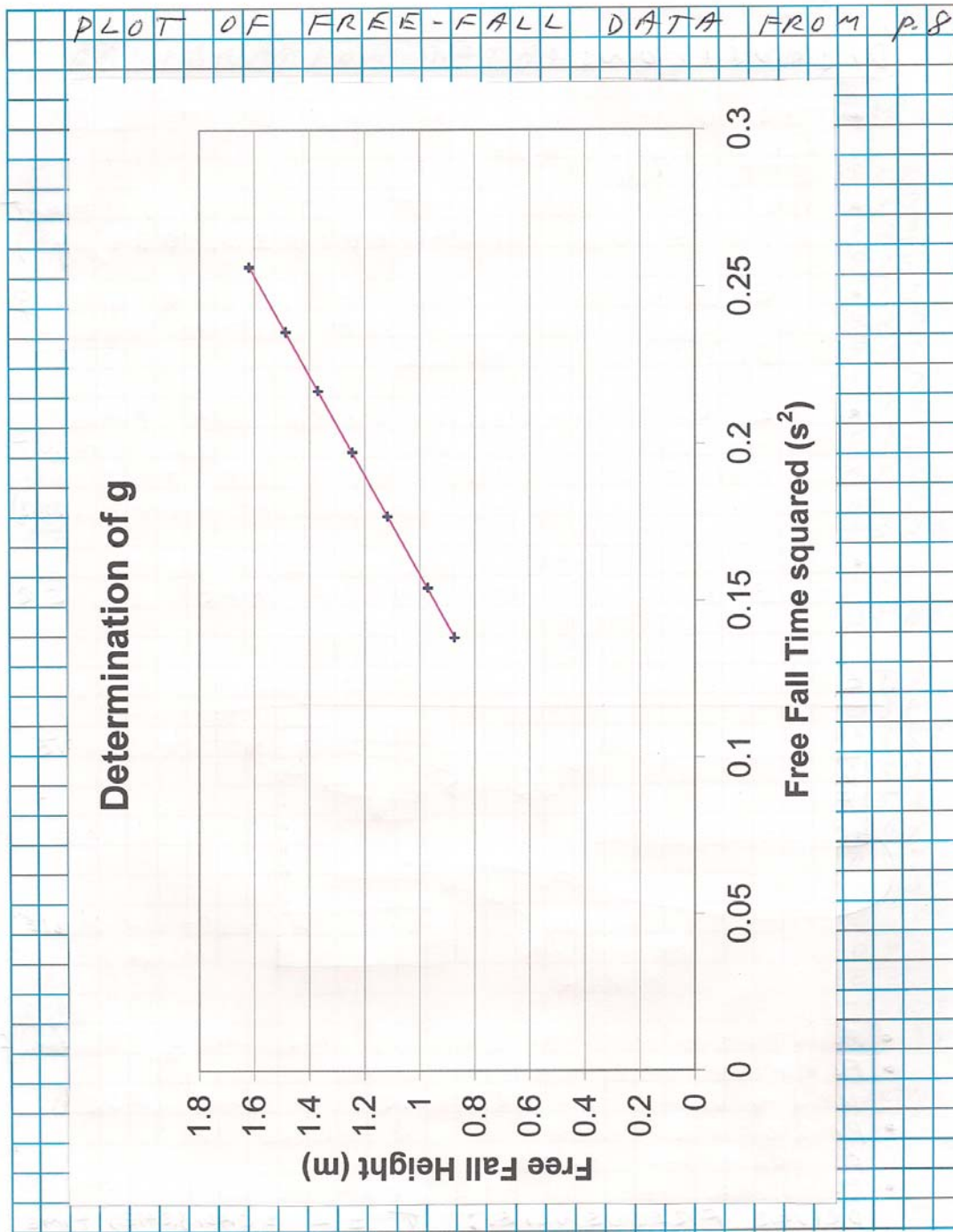
Example Laboratory Notebook Page 8

The key results from the seven data sets are summarized here, with units and uncertainties. A Least-Squares fit to the data (plotted on the next page) shows a good fit, with an intercept consistent with zero, but a slope which gives a strange result for g .



Example Laboratory Notebook Page 9

A plot of the data from the previous page. Use of cross-references is always helpful, especially if the reference is to information on some page other than the facing page.



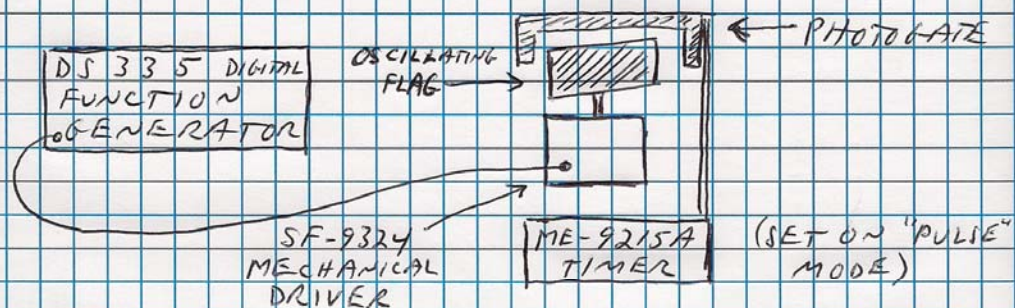
Example Laboratory Notebook Page 10

The experimental result for g does not agree with the published value. The experimenters begin to suspect a systematic error associated with their timing device, and prepare to perform a calibration routine, to correct the measured times.

DISCUSSION:

- THE QUALITY OF THE FIT IS GOOD ($r^2 = 0.56$)
 $\frac{1}{NA}$
- THE y -INTERCEPT IS CONSISTENT WITH ZERO ($a = -0.0018 \pm 0.0036 \text{ m/s}^2$)
- THE SLOPE GIVES A VALUE OF g IN POOR AGREEMENT WITH THE PUBLISHED VALUE
- THE REPRODUCIBILITY OF THE TIME MEASUREMENTS IS CONSISTENT WITH THE PRECISION OF THE TIMER DISPLAY (σ_t IS A FEW $\times 0.1 \text{ ms}$)
- WE SUSPECT A PROBLEM WITH THE CALIBRATION OF THE TIMER

SETUP FOR TIMER CALIBRATION



- FUNCTION GENERATOR PRODUCES SINE WAVE @ FREQUENCY f
- FLAG ON MECHANICAL DRIVER OSCILLATES UP AND DOWN IN RESPONSE TO SINE WAVE
- PHOTOGATE 'TIMER' IS TRIGGERED BY OSCILLATING FLAG
- MEASURE PERIODS FOR SEVERAL DIFFERENT DRIVER FREQUENCIES: $T_{\text{CAL}} = \frac{1}{f} = \text{CALIBRATED TIME}$

Example Laboratory Notebook Page 11
Data for timer calibration.

6 MAY 2010

CALIBRATION DATA: ($\sigma_f = 10^{-6} \text{ Hz}$)			
$f = 1.5 \text{ Hz}$	$f = 1.75 \text{ Hz}$	$f = 2.0 \text{ Hz}$	$f = 2.25 \text{ Hz}$
TIMES (s)			
0.5866	0.5044	0.4414	0.3914
0.5853	0.5046	0.4404	0.3917
0.5884	0.5016	0.4414	0.3923
0.5877	0.5028	0.4393	0.3915
0.5880	0.5018	0.4414	0.3913
0.5898	0.5040	0.4406	0.3927
0.5856	0.5048	0.4399	0.3903
0.5885	0.5026	0.4406	0.3917
0.5870	0.5047	0.4393	0.3912
0.5862	0.5024	0.4408	0.3935
MEAN 0.58731	0.50337	0.44051	0.39176
ST. DEV 0.00142	0.00126	0.00080	0.00089
σ/μ 0.00045	0.00040	0.00025	0.00028

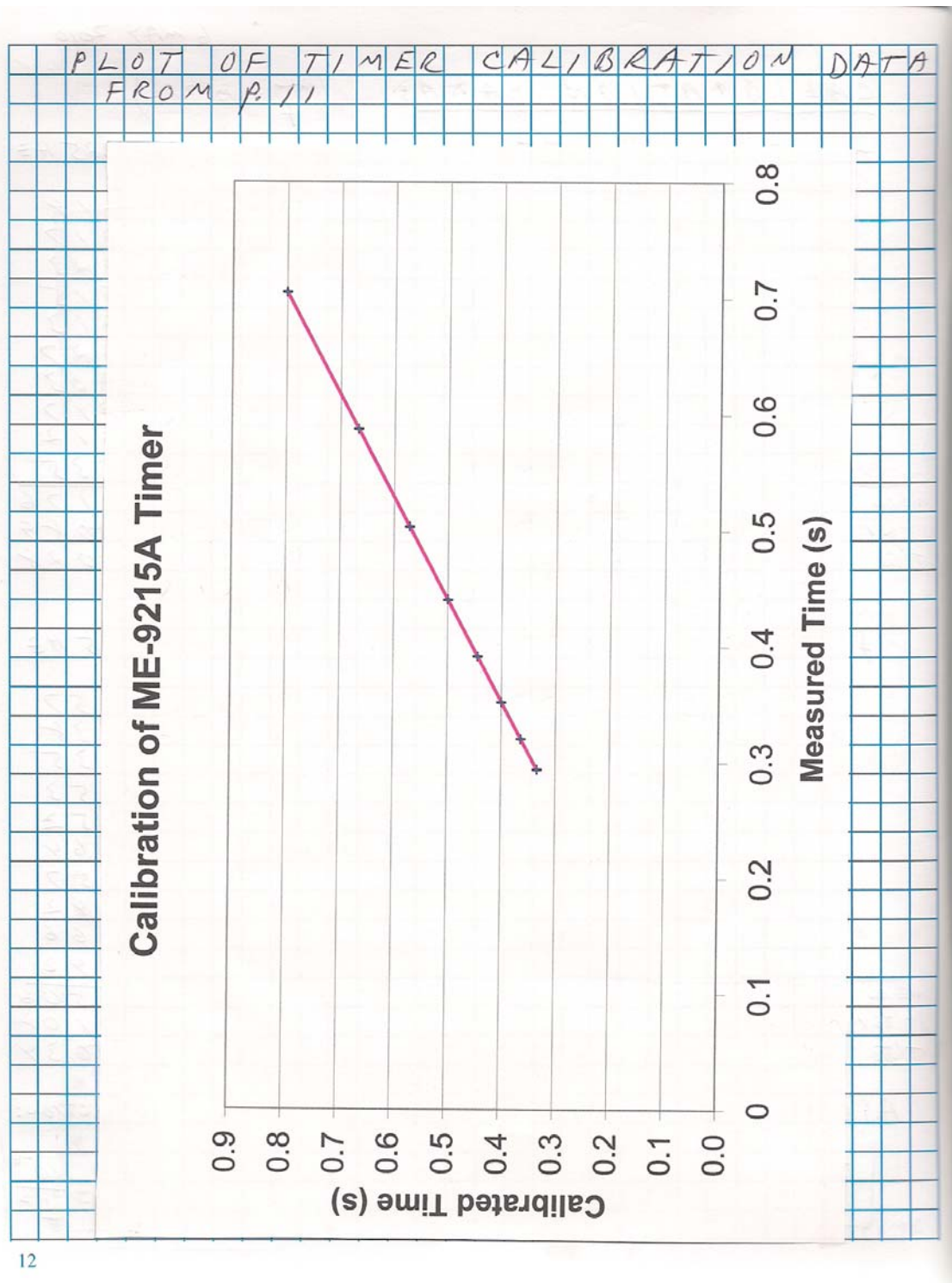
$f = 2.5 \text{ Hz}$	$f = 2.75 \text{ Hz}$	$f = 3.0 \text{ Hz}$	$f = 3.25 \text{ Hz}$
TIMES (s)			
0.3518	0.3200	0.2937	0.7057
0.3523	0.3203	0.2964	0.7038
0.3528	0.3201	0.2936	0.7038
0.3528	0.3208	0.2942	0.7035
0.3534	0.3204	0.2921	0.7053
0.3522	0.3205	0.2933	0.7052
0.3534	0.3204	0.2920	0.7064
0.3521	0.3206	0.2949	0.7057
0.3521	0.3209	0.2961	0.7042
0.3507	0.3205	0.2950	0.7066
0.3523	0.3211		0.7042
MEAN 0.35231	0.32051	0.29413	0.70506
ST. DEV 0.00079	0.00033	0.00150	0.00108
σ/μ 0.00025	0.00010	0.00047	0.00034

ASSUME LINEAR CALIBRATION FUNCTION:

$$t_{\text{CAL}} = a + b t_{\text{meas}} \quad \left| \quad t_{\text{CAL}} = \frac{1}{f} \quad \left| \quad \sigma_{t_{\text{CAL}}} = \frac{\sigma_f}{f^2} = \frac{10^{-6}}{f^2} \right. \right.$$

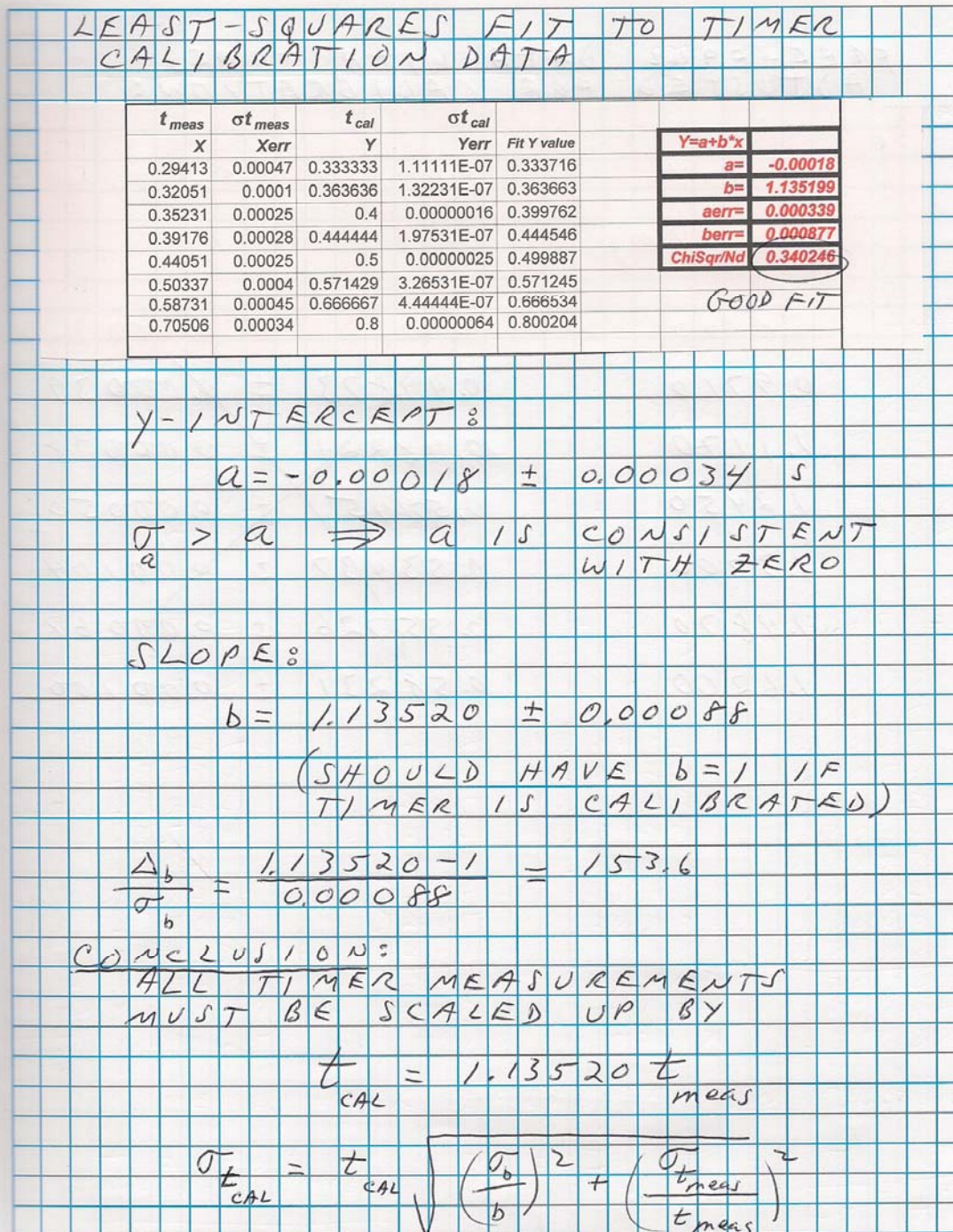
Example Laboratory Notebook Page 12

Plot of timer calibration results, with cross-reference to data page.



Example Laboratory Notebook Page 13

Results of a Least-Squares fit to the calibration data indicate the need to multiply all of the measurements by a constant factor. Since the calibration scale factor has some uncertainty, this will increase the uncertainty in the measured times, as shown by the error propagation equation at the bottom of the page.



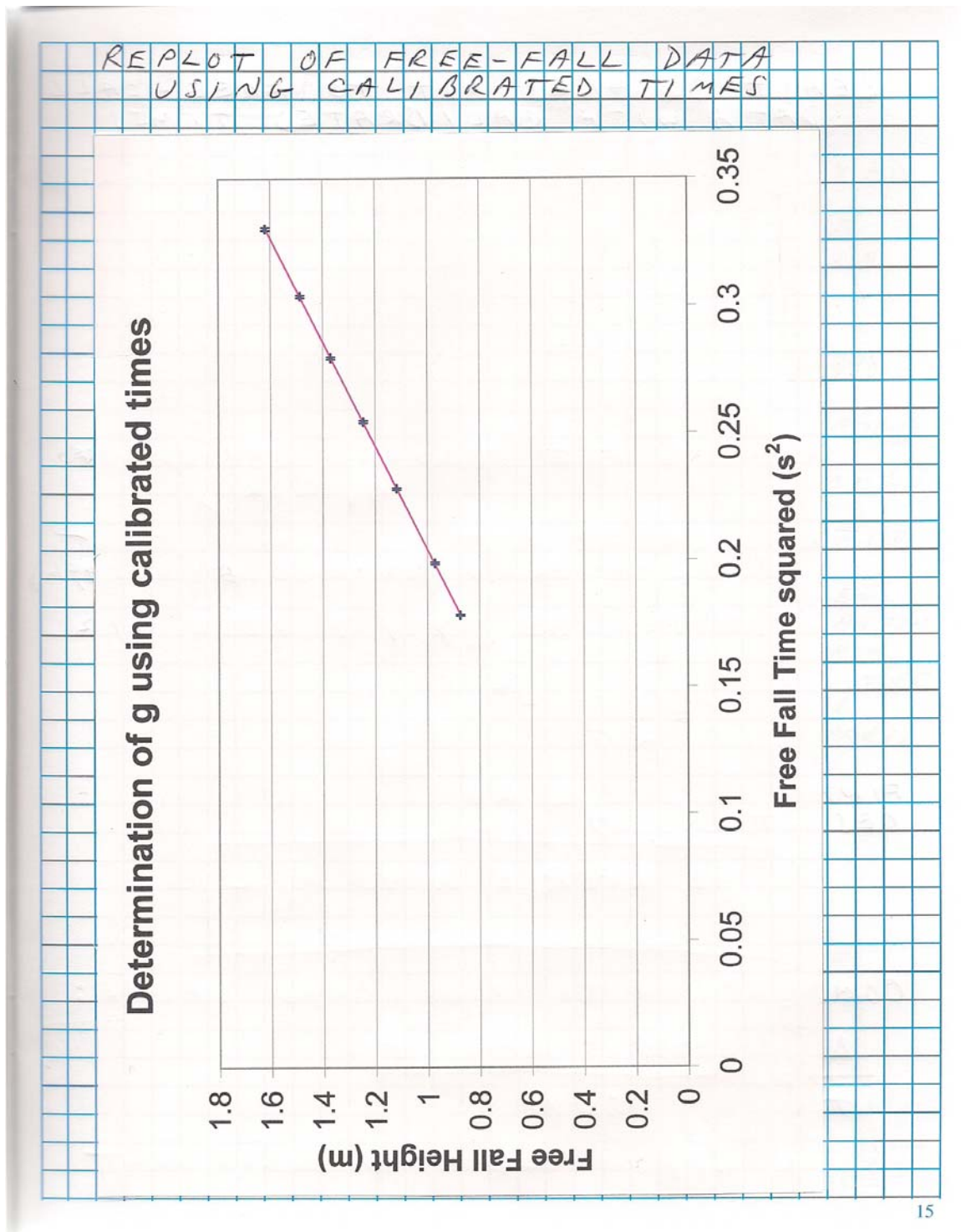
Example Laboratory Notebook Page 14

The calibration scale factor is applied to the original free-fall time results, and the corrected times and uncertainties are shown for each height.

FREE-FALL DATA WITH TIMES ADJUSTED FOR CALIBRATION	
HEIGHT ($\pm 0.0014\text{ m}$)	CALIBRATED TIMES (s)
0.8740	0.42220 ± 0.00040
0.9710	0.44580 ± 0.00050
1.1170	0.47757 ± 0.00043
1.2450	0.50440 ± 0.00050
1.3700	0.52872 ± 0.00045
1.4870	0.55116 ± 0.00053
1.6200	0.57477 ± 0.00053

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The free-fall data are replotted, after having been corrected for the timer calibration.



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A Least-Squares fit to the corrected data gives a value of g in good agreement with the published value.

LEAST-SQUARES FIT TO FREE-FALL DATA WITH CALIBRATED TIMES

t^2	$\sigma_{t^2} = 2t\sigma_t$	y	σ_y	Fit Y value	
0.178256	0.000336351	0.874	0.0014	0.872579	
0.198742	0.000447042	0.971	0.0014	0.973041	
0.228071	0.000409233	1.117	0.0014	1.116869	
0.254423	0.000501199	1.245	0.0014	1.2461	
0.279544	0.000479042	1.37	0.0014	1.369295	
0.30378	0.000579632	1.487	0.0014	1.488147	
0.330366	0.00061438	1.62	0.0014	1.618523	

$Y=a+b*x$	
$a=$	-0.001583
$b=$	4.903978
$aerr=$	0.004941
$berr=$	0.020122
$ChiSqr/Nd$	0.319191

GOOD FIT

- GOOD FIT (REDUCED CHI-SQUARED $\chi^2_{red} = 0.32$)
- Y-INTERCEPT IS CONSISTENT WITH ZERO ($a = -0.0015 \pm 0.0049 m$)
- SLOPE: $b = 4.904 \pm 0.020 m/s^2$

$$y = \frac{1}{2} g t^2$$

FINAL RESULT: $g = 2b$

$$g = 9.808 \pm 0.040 m/s^2$$

COMPARISON TO PUBLISHED VALUE:

$$\frac{\Delta}{\sigma} = \frac{9.808 - 9.80118}{0.040}$$

$$= 0.17 \Rightarrow \text{GOOD AGREEMENT}$$

Example Laboratory Notebook Page 17

Discussion and Conclusions: the last page of notes for this experiment. The specific accomplishment is restated (good agreement with the published value for g). The original assumptions (constant acceleration; neglect air resistance) are revisited, and determined to be a valid assumptions.

DISCUSSION/CONCLUSIONS:

HAVING DISCOVERED AND FIXED THE PROBLEM WITH TIMER CALIBRATION, WE OBTAINED A VALUE FOR THE ACCELERATION DUE TO GRAVITY OF

$$g = 9.808 \pm 0.040 \text{ m/s}^2$$

THIS IS IN GOOD AGREEMENT WITH THE PUBLISHED VALUE FOR PITTSBURGH, PA (OUR LOCATION)

WE HAD ASSUMED THAT WE COULD NEGLECT AIR RESISTANCE.

OUR DATA INDICATE THAT THIS WAS A GOOD ASSUMPTION.

IF AIR RESISTANCE HAD BEEN SIGNIFICANT, THE ACCELERATION WOULD HAVE BEEN NONUNIFORM, AND THE RESULTING FIT OF THE DATA TO $y = \frac{1}{2}gt^2$ WOULD HAVE YIELDED A LARGE CHI-SQUARED.

1.6 Laboratory Notebook Checklist

Organizational details:	
Use section headings, such as “Purpose”, “Theory”, “Diagram”, “Data”, etc	
No loose sheets of paper. (Staple or tape each graph onto a <u>blank</u> page.)	
Log entries dated for each day of work	
Purpose:	
State the goal or hypothesis of the experiment.	
Theory:	
Show the key physics equation that motivates the experiment.	
Define all the symbols in the equation.	
Method:	
Describe what measurements will be made and how will the data be analyzed	
Diagram:	
Schematic diagram with components labeled and dimensions indicated	
Original Data:	
All required data recorded by hand in the lab notebook	
Data tables with column headings, units and uncertainties	
Cross-reference to page where data are plotted (if not on the facing page)	
Data Analysis (Graphs):	
Data points clearly visible	
Error bars shown on data points	
“Best Fit” computer-generated line or curve superimposed on data	
Axes labeled with physical quantity and units of measurement	
Descriptive title indicating purpose of the plot (not just “y vs. x”)	
Cross-reference to data page (if the data are not on the immediately facing page)	
Discussion of Fitting Parameters ($a \pm \sigma_a$, $b \pm \sigma_b$ chi-squared)	
Is the fit good? (Is the reduced chi-squared approximately equal to one?)	
Is the intercept, a, consistent with the expected intercept?	
Does the slope, b, (or result derived from the slope) agree with prediction?	
Data Analysis (Equations):	
All equations used to analyze the data must be shown, with symbols defined	
Data Analysis (Error propagation):	
Error analysis equations must be shown with appropriate symbols	
Summary Table of Important Final Results	
Round off the uncertainty in each result to two significant digits.	
Write the result in same format (power of ten and decimal places) as uncertainty.	
Show the units (SI system preferred)	
Include the predicted or published value for comparison	
Calculate Δ/σ (difference between experiment and prediction, over uncertainty)	
Discussion/Conclusions	
State clearly whether results agree with predictions, or not.	
Re-state the key physics: what do your results tell you about the physical system?	
If results do not agree with prediction, give plausible explanation.	

Treatment of Experimental Errors

2.1 Sources of error in measurement

Every quantitative experiment by definition involves the activity of measurement. It is quite reasonable to believe that the quantity being measured has some 'true' value. However, due to the limitations of any measuring device, or technique of measuring, we must admit that we do not know with absolute certainty just what the true value is. For example, when using a meter stick to measure the amplitude of oscillation of a mass on a spring, the inherent precision of the scale, having 1 mm divisions, may be further limited by problems of parallax, judgment, etc. Similarly, the precision of a stopwatch used to measure the period of oscillation of the same mass-spring system is likely to be limited by the reaction time of the person using it. Additional error may be introduced if, for example, the meter stick being used had been calibrated at quite a different temperature from that at which the experiment is run and is thus no longer the same length due to thermal expansion. Or perhaps the stopwatch does not reset all the way back to zero, thus always yielding a measurement which is slightly too large.

We begin to see that the experimental errors can be divided into two distinctly different parts. **Systematic errors** result from factors which tend to cause reproducible inaccuracies, such as faulty equipment, calibration, or technique, or recurring fluctuations in the environment, such as diurnal temperature variation. **Random errors**, by contrast, are equally likely to cause the results of repeated measurements to be larger or smaller than the true value, and arise due to unpredictable factors such as reaction time, random fluctuations in the environmental conditions during the course of the experiment, or non-uniformities in the running of mechanical parts of the apparatus.

Systematic errors can often be reduced by careful calibration of the equipment before the actual experimental measurements are made. Random errors, on the other hand, can never be eliminated completely, but can be minimized in two ways that will be explained in more detail shortly.

Of course, there are other sorts of error, namely **personal error**, or outright mistakes. However, these will not be treated here except to say that they can usually be avoided by use of careful technique and the exercise of common sense. Should they occur anyway, they are likely to show up in the analysis stage as outliers: data points which are far from the predicted curve in comparison to other similar measurements.

2.2 Random errors and the normal probability distribution

In order to gain a better understanding of the nature of random errors and ways of minimizing the experimental uncertainty in a measured quantity, we need to define some terms.

Given a set of N measurements, x_i ($i = 1, 2, \dots, N$) of a quantity x , the **mean** or average value of the set is defined as:

$$\bar{x} = \frac{1}{N} \sum_{i=1}^N x_i \quad 2.1$$

The **standard deviation** of the distribution is defined as:

$$\sigma = \left[\frac{1}{N-1} \sum_{i=1}^N (x_i - \bar{x})^2 \right]^{\frac{1}{2}} \quad 2.2$$

The standard deviation σ of a distribution of measurements is an estimate of the random error associated with each individual measurement, that is, the typical error that you would expect if you were to make just one measurement. In other words we can identify it as an estimate of the **uncertainty associated with a single measurement**. The magnitude of σ depends upon the precision of your experimental apparatus, and on the extent to which you are able to control the conditions of the experiment.

Using the Dart Game experiment as an example, we plot the measured quantity x on the horizontal axis in figure 2.1. Let us divide the scale of x into small, equal intervals Δx , called bins. The choice of bin size is somewhat arbitrary. In figure 2.1 we chose $\Delta x = 1$, since each dart throw was assigned an integer number. On the vertical axis, we plot the number n of values of x which belong to the given bin. This graph is called a histogram.

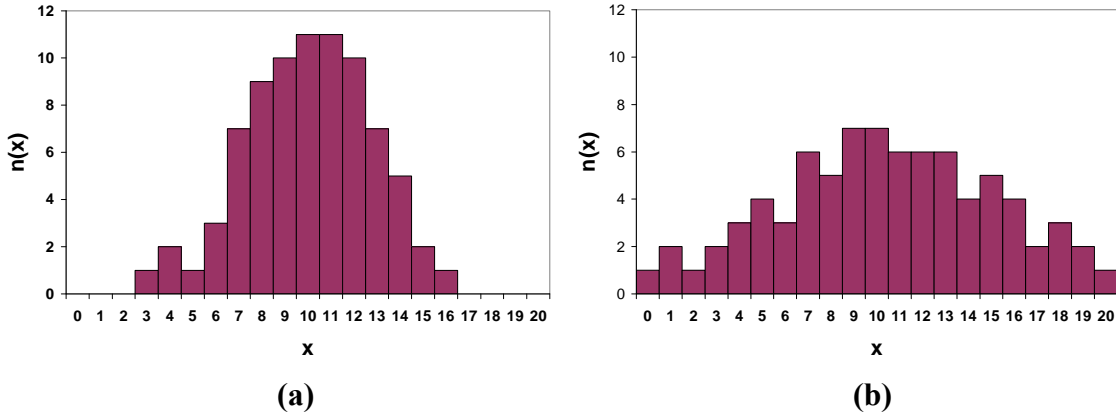


Figure 2.1: Histograms of the x -coordinate of darts thrown at a target centered at $x = 10$. A total of 80 darts were thrown by (a) a fairly skilled dart thrower, and (b) a not-nearly-so-skilled-as-the-dart-thrower-in-(a) dart thrower.

When the total number of data in the set N , obeys $N \gg 1$, the resulting histogram may start to look smooth, and sometimes approximates the so-called **normal distribution** curve (also called a **Gaussian distribution** or **Bell curve**). This distribution is defined by:

$$n(x) = \frac{N}{\sigma\sqrt{2\pi}} e^{-\frac{(x-\bar{x})^2}{2\sigma^2}} \Delta x \quad 2.3$$

The factor in front of the exponent is chosen in such a way that, if you integrate this expression over all the possible values of x , the result is the total number of measurements N . It is an empirical fact that the normal distribution curve is often a good representation of the random errors associated with measurements, and it is widely used for this reason.

To use equation 2.3, first calculate the mean \bar{x} , and the standard deviation σ for your set of measurements according to equations 2.1 and 2.2. Next, you choose a suitable bin size Δx . In our example of the darts experiment, the logical choice of bin size is $\Delta x = 1$, since the target is divided into columns labeled with successive integers. Having already plotted a histogram of your data showing the actual number of times a measurement occurred in each bin, you can calculate the expected number of measurements in each bin as predicted by equation 2.3, using your values of \bar{x} and σ . Finally, plot these calculated values for $n(x)$ on the same graph as the histogram to see how well your actual distribution of measurements can be described by the normal probability distribution. The normal distribution is not necessarily the correct probability distribution for a given experiment, but it is often a reasonable approximation of that distribution.

2.3 Random errors on the mean

In the preceding section we considered a set $x_1, x_2, \dots, x_i, \dots, x_N$ of N measured values of the quantity x . The mean value over this set was called \bar{x} . Suppose that we repeat the measurements, obtaining N' different sets, each consisting of N measured values. We label these sets by the subscript $j = 1, 2, \dots, N'$. This means \bar{x}_j over each of these N' sets will usually differ from each other. In other words, the mean over a set of measurements is affected by random error. By analogy with what we did for individual measurements in the last section, we can plot the various means \bar{x}_j on a histogram. Also, we can define the **mean of means** $\bar{\bar{x}}$:

$$\bar{\bar{x}} = \frac{1}{N'} \sum_{j=1}^{N'} \bar{x}_j \quad 2.4$$

and the **standard deviation of the mean** σ_M :

$$\sigma_M = \left[\frac{1}{N'-1} \sum_{j=1}^{N'} (\bar{x}_j - \bar{\bar{x}})^2 \right]^{\frac{1}{2}} \quad 2.5$$

The standard deviation of the mean σ_M is an estimate of the random error associated with the mean of a particular set of measurements, that is, the typical error that you would expect if you were to make just one set of N measurements and take the mean. In other words, we can identify it as an estimate of the **uncertainty associated with the mean**.

Probability theory can show that, provided $N \gg 1$,

$$\sigma_M = \frac{\sigma}{\sqrt{N}} \quad 2.6$$

From equation 2.6 we can see why it is advantageous to average over as many data as possible.

It is important always to make clear to which quantity each standard deviation applies. If we consider any single measurement of a quantity x , then **σ represents an estimate of the uncertainty associated with that single measurement**. If we consider the mean value \bar{x} of N measurements of x , then **σ_M represents an estimate of the uncertainty associated with that mean value**. It is good practice to always use subscripts to distinguish between the different standard deviations and to indicate to which value they apply.

2.4 Setting an upper bound when the random error is too small to measure

There may be occasions when the fluctuations in a measured quantity due to a random error are too small to be detected by your measuring device. If it happens that every repeated measurement yields exactly the same result, how do you estimate the experimental error in your measurement? In such instances, it would appear that the only thing that limits your determination of the actual value of the quantity being measured is the resolution of the measuring device itself. The following are some guidelines for setting an *upper bound* on the errors associated with such measurements.

- **An analog scale:** When reading an analog scale (e.g. a metric ruler or a moving-coil meter, etc.), an upper bound on the random error for a perfectly reproducible measurement may be estimated at $\pm 1/2$ of the finest division on the scale (or whatever fraction you consider to be reasonable, depending upon the physical size of the divisions).
- **A digital display:** When reading a digital scale, an upper bound on the random error associated with a perfectly reproducible reading may be estimated at $\pm 1/2$ of the least significant digit.

It should be understood, however, that this method *overestimates* the amount of random error. The actual random errors may be smaller. Also remember that there are systematic errors, which must be estimated separately.

2.5 Errors associated with statistical counting experiments

Finally we consider the error associated with any statistical counting experiment (e.g. the counting of events in a radioactive decay experiment).

In probability theory the **Poisson distribution** is a discrete probability distribution that expresses the probability of a number of events occurring in a fixed period of time if these events occur with a known average rate, and are independent of the time since the last event, as is the case with any spontaneous atomic or nuclear transition.

The error associated with counting events is simply \sqrt{N} where N is the total number of events counted. In such cases as these, it is always desirable to allow the experiment to run long enough so that $\sqrt{N} \ll N$. Errors associated with counting experiments are often referred to as **statistical errors**.

Example: If a sample emits a radioactive particle on average once per minute, and you are interested in the number of events occurring in a 16 minute interval, you would expect to obtain a count of 16 ± 4 .

2.6 Reducing experimental uncertainty

As mentioned previously, the standard deviation σ of a set of measurements is fixed by the quality and conditions of the experiment. Once again the experiment of throwing darts provides a good illustration of this point. See figure 2.1, above.

Note that while each of the histograms in figure 2.1 contain a total of 80 measurements, the width of distribution (a) (characterized by the standard deviation σ_a) is considerably smaller than the same for distribution (b). The mean value of distribution (a) is thus a more reliable measurement of the true, or ‘target’ value than the mean value of distribution (b).

There are **two ways to reduce the effects of random error**. The first, and perhaps most obvious way is to obtain more precise equipment and/or to improve your technique of measurement. The effect of this is to **reduce the standard deviation σ** of your measurements. The second way, suggested by equation 2.6, is simply to increase the number of repeated measurements N . In this case, there will be no effect on the standard deviation of the distribution, but it will **reduce the random error on the mean**.

2.7 The statistical significance of experimental results

We will now discuss the use of the normal probability distribution in determining the statistical significance of our results. Again we use the example of the darts experiment. We define the **absolute difference Δ** as the difference between the experimental result and the predicted result.

Consider the results of the dart throwing experiment (figure 2.1). The mean value for each set of data (equation 2.1) and the corresponding standard deviation of the mean (equation 2.6) are displayed below:

Predicted Result	Experimental Results	
	Trial (a)	Trial (b)
10.00 in	10.04 ± 0.30 in	10.42 ± 0.51 in

Table 2.1: Final results for the ‘dart-throwing’ experiment, indicating the predicted result and the mean value of N throws with the uncertainty on the mean.

Note that in the example above, the results are given with two significant figures in the uncertainty, following the convention discussed in section 1.5.

Examining table 2.1, we can see that for trial (a) the experimental result differs from the predicted result by $\Delta = 0.04$ inches. If we divide this difference by the associated uncertainty $\sigma = 0.30$ inches, we find that the result is only 0.13 times σ away from the predicted value. Similarly, the result for trial (b) is 0.8 times σ away from the predicted value. Both of these results are considered to be in good agreement with the predicted value since they both differ from the predicted value by less than one σ . In other words, in both cases the predicted value lies within the range of uncertainty of the experimental result.

If we use x_{pred} to indicate the predicted value of the quantity x , and σ to indicate the uncertainty in an experimental measurement, we can write an expression for the probability of obtaining an experimental value in the interval x to $x+dx$ (assuming the distribution is Gaussian and centered on the predicted value):

$$P(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x_{pred})^2}{2\sigma^2}} dx \quad 2.7$$

Given that an experimental result differs from the predicted value by a certain number times σ , the question arises, “How do I decide whether or not my experimental result is in agreement with the predicted value?” Unfortunately there is no well-established answer to this question. However, equation 2.7 can be used to provide a statistical answer.

Suppose, for example, that your experimental result differs from the predicted value by one σ or less. The probability of obtaining a result which differs from the predicted value by not more than one σ is found by integrating equation 2.7 from $x_{pred} - \sigma$ to $x_{pred} + \sigma$. (Fortunately standard math tables¹ are available which eliminate the need to do the integral.)¹

$$\int_{x_{pred}-\sigma}^{x_{pred}+\sigma} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x_{pred})^2}{2\sigma^2}} dx \approx 0.68269 \quad 2.8$$

This indicates a probability of approximately 0.68 of obtaining a result which differs from the predicted value by one σ or less. To put it somewhat differently, if you were to repeat the experiment, there is a 32% chance that your result would differ from the predicted result by one σ or more.

Similarly, the probability of obtaining a result which differs from the predicted value by not more than two σ is found by integrating equation 2.7 from $x_{pred} - 2\sigma$ to $x_{pred} + 2\sigma$:

$$\int_{x_{pred}-2\sigma}^{x_{pred}+2\sigma} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x_{pred})^2}{2\sigma^2}} dx \approx 0.95449 \quad 2.9$$

which indicates less than a 5% chance of missing the predicted value by more than 2σ .

¹ See, for example, Table C2 in Philip R. Bevington and D. Keith Robinson, Data Reduction and Error Analysis for the Physical Sciences 3rd edition, McGraw-Hill, 2003

And the integral of equation 2.7 from $\mu - 3\sigma_M$ to $\mu + 3\sigma_M$:

$$\int_{x_{pred}-3\sigma}^{x_{pred}+3\sigma} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{(x-x_{pred})^2}{2\sigma^2}} dx \approx 0.99730 \quad 2.10$$

indicates a probability of only 0.3% of missing the predicted value by more than 3σ .

We can make use of this information to establish a convention for deciding how well a measurement agrees with the predicted result:

1. Subtract your experimental result from the predicted result
2. Divide the difference by the uncertainty²

$$\frac{\Delta}{\sigma} = \frac{|\text{experimental result} - \text{predicted result}|}{\text{uncertainty}} \quad 2.11$$

3. Consult table 2.2

$\frac{\Delta}{\sigma} =$	then the agreement is ...
1 or less	good
between 1 and 2	fair
between 2 and 3	marginal
3 or more	poor

Table 2.2: A convention for establishing agreement between experimental and predicted results.

Note that the probability of obtaining a $\frac{\Delta}{\sigma}$ of 3 or more is only 0.3%. According to our convention this is unlikely enough to indicate a problem. There are three possibilities:

1. The predicted value is wrong.
2. Your experimental measurements are suffering from systematic error.
3. You have underestimated your uncertainty.

² This is the uncertainty associated with Δ . See section 2.8 to determine how to propagate the uncertainty associated with the experimental result and the uncertainty associated with the predicted result.

2.8 Propagation of Uncertainties

Most experimental work involves the calculation of the final result from two or more measured quantities. Thus it is necessary not only to determine the experimental uncertainty associated with the measured quantities but also to determine the experimental uncertainty associated with the final result.

We often perform measurements in which the results depend on several measured inputs, where these inputs may be measured with different precision. We want to know how the uncertainties on the measured quantities affect the uncertainty on our ultimate result.

First, let us consider a single measurement and the trivial case where our result *is* the directly measured quantity. For example, say we wish to know the diameter of the base of a cylinder and we directly measure this diameter to obtain $d \pm \sigma_d$. Clearly the uncertainty in our result R is simply the uncertainty associated with our measurement:

$$R \pm \sigma_R = d \pm \sigma_d \quad 2.12$$

Now let us consider a result that depends upon a function of our measured quantity. For example, say we wish to know the area of the base of the cylinder.

$$A = \pi \left(\frac{d}{2} \right)^2 \quad 2.13$$

In general, if the result is a function of our measured quantity, then the uncertainty in the result is the derivative of the result R with respect to the measurement m times the measured uncertainty.

$$\sigma_{result} = \left| \frac{dR}{dm} \right|_m \times \sigma_{measurement} \quad 2.14$$

For our example, the uncertainty in the area is determined by taking the derivative of A (equation 2.13) with respect to d , and multiplying by the uncertainty in diameter, so that

$$R \pm \sigma_R = A \pm \sigma_A$$

where

$$\sigma_A = \frac{\pi d}{2} \times \sigma_d \quad 2.15$$

We now get to the situation where more than one measured quantity can affect our result. Suppose we wish to find the volume of the cylinder.

$$V = \pi \left(\frac{d}{2} \right)^2 h \quad 2.16$$

Clearly our derivative formula is still relevant, but we need to find a way to add them together. In the case where the measurements are independent (they do not depend on each other), it can be shown that we can add the uncertainties with a "Pythagorean Theorem" like sum.

In general, given measured quantities A, B, \dots with known uncertainties $\sigma_A, \sigma_B, \dots$ respectively, if a result R is calculated as some general function $f(A, B, \dots)$ of the measured quantities: $R = f(A, B, \dots)$, then the uncertainty on R , σ_R , is:

$$\sigma_R = \left[\left(\frac{\partial f}{\partial A} \sigma_A \right)^2 + \left(\frac{\partial f}{\partial B} \sigma_B \right)^2 + \dots \right]^{\frac{1}{2}} \quad 2.17$$

where the ∂ symbol indicates the partial derivative of the function taken only with respect to one variable and treating the other variable(s) temporarily as being constant.

For our example, the result clearly depends on two independent measurements: the diameter of the base $d \pm \sigma_d$, and the height of the cylinder $h \pm \sigma_h$.

The uncertainty in the volume is then determined by taking the partial derivative of V (equation 2.16) with respect to d , and with respect to h , and combining them according to equation 2.17, so that

$$R \pm \sigma_R = V \pm \sigma_V$$

where

$$\sigma_V = \left[\left(\frac{\partial V}{\partial d} \sigma_d \right)^2 + \left(\frac{\partial V}{\partial h} \sigma_h \right)^2 + \dots \right]^{\frac{1}{2}}$$

$$= \left[\left(\frac{\pi d h}{2} \sigma_d \right)^2 + \left(\frac{\pi d^2}{4} \sigma_h \right)^2 \right]^{\frac{1}{2}} \quad 2.18$$

A Consistency Check

Just to reassure you that the rule for propagation of uncertainties is consistent with our discussion of mean, standard deviation and uncertainty on the mean, consider the uncertainty associated with the mean value of a distribution of measurements A_i , each of which has some experimental uncertainty σ_{Ai} . The mean is given by:

$$\bar{A} = \frac{1}{N} \sum_{i=1}^N A_i \quad 2.19$$

We may obtain the uncertainty on \bar{A} by applying the general rule for propagation of uncertainties (equation 2.17):

$$\sigma_{\bar{A}} = \left[\sum_{i=1}^N \left(\frac{\partial \bar{A}}{\partial A_i} \sigma_{Ai} \right)^2 \right]^{\frac{1}{2}} = \frac{1}{N} \left[\sum_{i=1}^N (\sigma_{Ai})^2 \right]^{\frac{1}{2}} \quad 2.20$$

If all of the individual σ_{Ai} are the same, that is, if we have the same experimental uncertainty on each of the individual measurements (which is the case if we take the standard deviation to be the typical error on a single measurement), then we are left with:

$$\sigma_{\bar{A}} = \frac{1}{N} (N \sigma_A^2)^{\frac{1}{2}} = \frac{\sigma_A}{\sqrt{N}} \quad 2.21$$

Thus, if the standard deviation σ_A of a set of measurements of the quantity A is taken to be the typical uncertainty associated with making any single measurement of A , then, by applying the rule for the propagation of uncertainties, we find that the propagated error associated with the mean value \bar{A} is $\sigma_{\bar{A}} = \sigma_A / \sqrt{N}$, which is identical to the uncertainty on the mean given by equation 2.6.

As an exercise: Some commonly used examples of error propagation calculations are summarized on the next page. Derive the result for each of these examples by applying the general rule for the propagation of uncertainties (equation 2.17).

2.9 Summary of rules for propagating uncertainties

Given measured quantities A, B, \dots with associated random uncertainties $\sigma_A, \sigma_B, \dots$ respectively, i.e. $A \pm \sigma_A, B \pm \sigma_B, \dots$:

General Function of One Variable:

$$\text{If } C = f(A) \text{ then } \sigma_C = \left| \frac{df}{dA} \right| \sigma_A$$

Examples:

$$\text{Multiply by a constant: } \text{If } C = nA \text{ then } \sigma_C = |n| \sigma_A$$

$$\text{A Power: } \text{If } C = A^m \text{ then } \sigma_C = \left| m \frac{C}{A} \right| \sigma_A$$

$$\text{Logarithm: } \text{If } C = \ln(A) \text{ then } \sigma_C = \frac{1}{A} \sigma_A$$

$$\text{Inverse sine function: } \text{If } C = \sin^{-1}(A) \text{ then } \sigma_C = \frac{1}{\sqrt{1-A^2}} \sigma_A$$

$$\text{Inverse tangent function: } \text{If } C = \tan^{-1}(A) \text{ then } \sigma_C = \frac{1}{1+A^2} \sigma_A$$

General Function of Two or more Variables:

$$\text{If } C = g(A, B, \dots) \text{ then } \sigma_C = \left[\left(\frac{\partial g}{\partial A} \sigma_A \right)^2 + \left(\frac{\partial g}{\partial B} \sigma_B \right)^2 + \dots \right]^{\frac{1}{2}}$$

where the ∂ symbol indicates the *partial derivative* of the function with respect to one variable only (the other variables being treated temporarily as constants).

Examples:

$$\text{Sum: } \text{If } C = A + B \text{ then } \sigma_C = \sqrt{\sigma_A^2 + \sigma_B^2}$$

$$\text{Difference: } \text{If } C = A - B \text{ then } \sigma_C = \sqrt{\sigma_A^2 + \sigma_B^2}$$

$$\text{Product: } \text{If } C = A \times B \text{ then } \sigma_C = C \left[\left(\frac{\sigma_A}{A} \right)^2 + \left(\frac{\sigma_B}{B} \right)^2 \right]^{\frac{1}{2}}$$

$$\text{Quotient: } \text{If } C = \frac{A}{B} \text{ then } \sigma_C = C \left[\left(\frac{\sigma_A}{A} \right)^2 + \left(\frac{\sigma_B}{B} \right)^2 \right]^{\frac{1}{2}}$$

Note: The above rules hold only if the uncertainties on A, B , etc. are *uncorrelated*. That is, the deviation of B from its true value is random and independent of the deviation of A from its true value during the same measurement.

Fitting a Straight Line to a Set of Data by the Method of Least Squares

3.1 Introduction

In many of the experiments in this course you will discover a linear relationship between two physical quantities. Such a relationship may be written, in general as

$$y = a + bx \quad 3.1$$

According to equation 3.1, a graph of y versus x yields a straight line with intercept a and slope b . One of the goals of the experiment may be to determine a third physical quantity which is related somehow to the slope b of the plot of y vs. x . The approach is to make measurements of the quantities y and x , plot a graph of the data, and then find the best possible straight-line relationship between these two quantities. A standard technique for finding the best fit to the data is the method of least squares. The name of the technique derives from the process of minimizing the sum of the squares of deviations between the actual data and the function which fits the data. The rationale for this process can be developed as follows.

3.2 The idea of least square fitting: Maximizing likelihood, minimizing χ^2

Suppose you have made a set of measurements $\{x_i, y_i \pm \sigma_i\}$, and would like to discover the function $f(x)$, which correctly describes the physical relationship between y and x . (Although the technique can be generalized to include the uncertainty in the quantity x , we have assumed, for the sake of simplicity, that it can be neglected.) If the measurements are distributed according to a Gaussian distribution, the probability p_i of obtaining any individual measurement y_i is:

$$p_i \propto e^{-\frac{(y_i - f(x_i))^2}{2\sigma_i^2}} \quad 3.2$$

The probability P of obtaining the entire set of measurements is found by multiplying the individual probabilities given by equation 3.2:

$$P \propto \prod_{i=1}^N e^{-\frac{(y_i - f(x_i))^2}{2\sigma_i^2}} = e^{-\sum_{i=1}^N \frac{(y_i - f(x_i))^2}{2\sigma_i^2}} \quad 3.3$$

Without the factor of 2 in the denominator of the exponent, equation 3.3 defines the so-called likelihood function:

$$L = e^{-\sum_{i=1}^N \frac{(y_i - f(x_i))^2}{\sigma_i^2}} \quad 3.4$$

The objective of any fitting process is to find the function $f(x)$ which maximizes the likelihood that the data are described by that function.

The exponentiated sum in equation 3.4 is given the name χ^2 (chi-square):

$$\chi^2 = \frac{(y_i - f(x_i))^2}{\sigma_i^2} \quad 3.5$$

Clearly, maximizing the likelihood L is equivalent to minimizing χ^2 . In general this must be done numerically. The standard approach involves an iterative process of guessing values for the parameters in the function $f(x)$, and calculating χ^2 until a minimum is found. The uncertainty in each one of the parameters is found by varying each parameter away from the best fit value, while re-optimizing all the other parameters in the fitting function, until the value of χ^2 increases by 1.

For the special case of a linear function, as given by equation 3.1, the parameters a and b , which minimize χ^2 , as well as the uncertainties in these parameters, can be calculated directly³. The solution is obtained in a straightforward way by taking partial derivatives of equation 3.5 with respect to the parameters a and b , setting these two derivatives equal to zero, and solving the pair of simultaneous equations.

3.3 Goodness of fit and the reduced χ^2

Recall from the discussion of the Gaussian distribution in chapter 2, that the standard deviation σ represents the typical difference between a measurement and the expected value. An examination of equation 3.5 suggests that the value of χ^2 should be approximately equal to the number of data points N , since it is just the sum of N terms, each of which is expected to be approximately equal to 1. However, if you recognize that a straight line provides an exact fit to any two data points (i.e. the deviations between each of the points and the line are identically zero), and a second order polynomial provides an exact fit to any three data points, etc., you might begin to suspect that the value of χ^2 is likely to be somewhat less than the number of data points for a good fit. We can define the number of degrees of freedom ν to be the number of data points N less the number of parameters p in the fitting function:

$$\nu = N - p \quad 3.6$$

The expected value of χ^2 is just the number of degrees of freedom ν . Assuming that the uncertainties σ_i on the individual data points have been measured properly, we can judge the *goodness of fit* by calculating the *reduced χ^2* . That is, χ^2 divided by the number of degrees of freedom. A good fit should have a *reduced χ^2* of approximately 1.

³ See, for example, Chapter 6 in Philip R. Bevington and D. Keith Robinson, Data Reduction and Error Analysis for the Physical Sciences 3rd edition, McGraw-Hill, 2003

3.4 Determining the best fit parameters using the least squares fitting program

In this course, most of the labor of minimizing χ^2 is done by means of a computer program, called LSF (Least Squares Fit). LSF is a Microsoft Excel workbook, written by Yi-Kuang Liu for the undergraduate physics laboratories (March 1997), and available on all the computers in the labs computer cluster. The basics for using this program are as follows:

- The user enters the set of measurements $\{x \pm \sigma_x, y \pm \sigma_y\}$ in four columns labeled **X**, **Xerr**, **Y**, **Yerr**. If the uncertainties in the x quantity are zero, or negligible, they may be omitted.
- When all of the data have been entered, the user clicks one of two options: **Yerr only**, or **X and Yerr**, depending on whether or not the uncertainty in the x quantity is to be included in the calculation.
- The results of the fit are five important pieces of information: the best fit intercept a , and the uncertainty on a , the best fit slope b , and the uncertainty on b , and the value of the reduced χ^2 (which is labeled as “Chisq/Nd” in the output). The results will appear in a table in the following format:

Y=a+b*x	
a=	
b=	
aerr=	
berr=	
ChiSqr/Nd	

- It is essential that you record all five pieces of information in your lab notebook each time you do a fit with the program.

You will receive specific training in the use of this program as part of the course. Additional discussion of least squares fitting and exercises involving χ^2 may be provided at the discretion of the instructor.

4

Reporting the Results

It is appropriate occasionally to do an experiment simply to satisfy one's own curiosity. However, the majority of scientific research should be done in order to contribute to the body of knowledge in the field. This necessarily requires communication of the results in some form, usually written (for publication in a professional journal), or in a more visual format (for presentation as a poster in a seminar or conference).

In a laboratory course, the chief aim of a report is not to show your instructor that you have covered the material and understood it, nor is it to see how well you can repeat known information from some reference. Rather it is to present in a thoroughly convincing and clear fashion the nature of your experiment and what can be concluded from the actual experimental observations. Most often, the report of a professional researcher is presented to one's peers. As you prepare your report, keep in mind that you are not writing a textbook on the subject (i.e. keep the report brief and to the point), nor are you writing to impress your instructor. Write or speak as you would in order to explain the result to your peers.

The internal organization of a report is not bound by any fixed rules, but will naturally vary, depending both on the style of the author and the experiment itself. However, one important point which applies to both written and oral reports *is* worth stating as a rule:

The report must stand on its own as a complete description of the experiment and the result.

It is not acceptable to ever write in a report, "See the lab notebook for details." All important figures, graphs, and numerical results (except for raw data) must be reproduced in the report itself.

4.1 The Elements of a Formal Written Report

The report which you submit should emphasize what *you* have done in your experiment. The most vital part of your report is the analysis of your results and the conclusions that you draw from them. The report should not include an exhaustive discussion of the theory, rather, you should include just enough theoretical background information (i.e. discussion of the equations relevant to your experiment) for the reader to be able to understand the physical system which you have investigated.

Remember that clear organization, complete sentences, good grammar, and spelling are required. In short, good English usage is essential to a good report. The following is a generic outline for a formal written report.

Title Page

The choice of format is left to the student. However, the following items must be included: title, author, date, and partner's name.

Abstract

The abstract is intended to capture the attention of the reader and to convince him/her that the rest of the report is worth reading. You should begin with a one or two word sentence description of the physical system which you investigated. Then state in another sentence or two what important measurements had to be made. Finally, in two or three sentences, state the key results and the conclusions that you were able to draw from them.

Since this is a course in experimentation, you should emphasize what you have discovered about the behavior of the real physical system that you studied. You should not include details of the theory in the abstract. However, it is appropriate to comment on how well your results agreed with the theory or with previously published results.

Your goal in writing an abstract is to be as informative, yet as brief as possible. Two sentences would be insufficient to convey the important information, while half a typewritten page is probably too long. For experiments in this course, one paragraph (150 words or less) should be enough to capture the essence of what you accomplished.

Dos and Don'ts for writing an abstract

- Do emphasize the important physics of the experiment.
- Use words, rather than equations.
- Do tell whether or not your result supports accepted theory or a previously published result.

- Do not include details of procedure, except to convey the essence of what you did.
- Do not refer to your work as "this lab." (That's so high school.)
- In general, do not include numerical results in an abstract (unless the entire experiment leads to just one single numerical result).

Body of the Report

The organization of the main body of the report is flexible, but it should contain the following sections:

- I. Introduction (What is the motivation for doing the experiment?)
- II. Apparatus and Procedure (How did you make the measurements?)
- III. Analysis (What did you do with the raw data to get the results?)
- IV. Results (Summarize and discuss the important final results.)
- V. Conclusions (What did you learn about the physical system?)

Next we will discuss the various parts of the body of the report in greater detail. Keep in mind that the most interesting and vital parts of the report are the last two items above: What are the results and what do they mean? The rest of the report should lead up to this.

I. Introduction

This section should make it clear what the experiment is about. Describe the physical system which you are investigating and tell what results you hope to achieve. Introduce the important equations which predict the behavior of the system. Each equation must be numbered sequentially in the margin of the report so that you can refer back to it later as needed.

II. Apparatus and Procedure

In this section you describe the experimental setup and tell how the measurements were made. But keep in mind that **you are not writing an instruction manual**. Do not tell the reader step-by-step what to do. Rather, describe what you did, using just enough detail to get the main points across. Include a carefully drawn schematic diagram of the apparatus. The diagram must have a figure number and a brief descriptive caption. In the text of the report, you will refer the reader to the figure and describe the important function of each piece of apparatus, but do not go into detail about how you constructed the apparatus. In other words, put the emphasis on the physics of the experiment, and not how the clamps and rods are put together or what knobs you have to turn.

In the Introduction section, you told the reader what important measurements had to be made and why. In the Apparatus and Procedure section, you must convince the reader that your experimental design really does enable you to make these desired measurements.

III. Results and Discussion

In the previous section you explained how the raw measurements were made. You must now guide the reader convincingly through the process of reducing the raw data in order to obtain the final results.

With reference to the equations that you presented in the introduction, you should describe to the reader how the data was analyzed. In the example formal report (page 50) the author references the introduction to show how his measurements (free-fall heights and times) are related to his final result (acceleration due to gravity).

You should not include raw data in your report. However important graphs that were used to analyze your data must be included (with figure number and brief descriptive caption) and must be specifically referenced in the text of the report. Graphs should not include such distracting information as slope calculations, written comments

(except captions), or annotations from the grader of your lab notebook. Graphs should always show error bars and a best-fit line.

Final results are usually presented in a table (with table number and brief descriptive caption) showing relevant physical variables, the result, previously published values or theoretical predictions, and the level of agreement. Tables must include uncertainties and proper units.

The Results section is the climax of your report. This is what the rest of the report has been leading up to. Now you are ready to compare your results critically with the theory, to support it, or demolish it, or modify it. If you have not already done so elsewhere, critically compare the actual experimental conditions with the assumptions of the theory. If, for example, the theory assumes no friction, is there anything in your experimental results which might indicate that this is a poor assumption? Be as specific as possible.

There are two extremes which you should avoid in discussion of your final results. The first is giving too little thought to sources of systematic error, and the second is dwelling too much on sources of error. If your results generally disagreed with predictions, make a serious effort to identify the most likely source of error. Do not invoke such ill-defined effects as ‘human error’ or ‘equipment error’ without offering any more thoughtful explanation. On the other hand do not make sources of error the main focus of your discussion. Be sure to look for such things as internal consistency of your results and qualitative agreement with the expected behavior of the system, as well as quantitative agreement, and point these out in the discussion.

Dos and Don'ts for Result and Discussion

- Do include all relevant graphs
- Do include uncertainties on all measured values
- Do include a table of final results

- Do not include raw data
- Do not include LSF spreadsheets
- Do not include error propagation formula

IV. Conclusions

While the Results section is the real climax of your report, it is important to leave the reader with a clear picture of what you have accomplished in your research. In the Conclusions section you reiterate (briefly) what you have set out to do, and state how well you have succeeded. You should emphasize the things that went right in your experiment, while being honest about results which do not agree with predictions. This part of the report is an excellent place to show the reader that you have a really sound understanding of the experiment you have performed.

Miscellaneous items

Two other organizational items remain to be discussed: the references and appendices.

- **References**

In references to literature throughout your report, use consecutively numbered footnotes placed in a list at the end of the report. Please follow the style used in Physical Review and illustrated below and on page 50 in the example formal report.

1. U. Regge and D. Zawischa, Phys. Rev. Lett. **61**, 149 (1988)
2. J. B. Marion, *Classical Dynamics* (Academic Press, 1970), p. 159.

- **Appendices**

The purpose of an appendix is to include some information which is relevant to the report, but would interrupt the flow of the discussion, or otherwise break up the structure and organization of the report. All appendices must actually be referred to in the main body of the report by saying, for example, “See Appendix A.” It is not appropriate to simply staple a bunch of pages to the end of the report as an afterthought and label them as an appendix. The following may be included as appendices in a report.

- Any discussion of a small point which is off the main train of thought and which cannot be made brief. For example, experiments performed to calibrate measurement devices. (See page 53 in the example formal report)
- Long mathematical treatments, if you need them to augment the theoretical discussion.

A few important details

- In deciding how much or how little to write, assume the reader to be one of your intelligent class-mates who has had the same courses which you have had except this course. As a general guideline, two pages of written text is too short; thirty pages, total, is too long. Approximately ten pages, including the most important graphs and drawings is appropriate.
- By the time you write your report, you should have repeated any obviously faulty measurements. If you have not done something that is asked for in the write-up, go back and get the data you need or give a valid explanation for why you couldn't.
- Every measured and calculated value must be quoted with its uncertainty unless a reason is given for not doing so.
- Present final results and comparisons to theoretical predictions or previously published values in a table. See, for example, table 1 in the example formal report (page 50). Be sure to use the proper number of significant figures following the convention discussed in section 1.4.
- **Graphs:** All graphs should have a figure number, title and brief descriptive caption. The title should not be, for example, 'Position vs. Time', but should indicate the purpose of the plot, for instance, 'Determination of the Velocity'. Choose sensible scales and mark data points clearly with their uncertainties. Label plots so that the page has to be turned by at most 90° , and only in the clockwise direction. Don't clutter graphs with calculations; these should be done in the text of your report. Always label the axes with the quantity measured and with units.
- **Keep your laboratory notebook up to date.** Doing a careful and thorough job of record keeping and analysis in the laboratory as you do the experiment may save you hours of work at home in preparing the report.

In the next section, an example formal report, based on the example laboratory notebook (section 1.5) is presented.

4.2 Example Formal Report

Free-Fall: Determination of the Local Acceleration Due to Gravity

Isaac Gnuton

Lab Partner: Janice Keppler

11 August 2010

We present the results of an experiment designed to determine the local acceleration due to gravity, g . Our technique involves dropping a steel ball through several measured distances, and measuring the corresponding times of fall. A plot of free-fall distance vs the square of the time yields a straight line, whose slope is $\frac{1}{2}g$. Our results indicate a value of $g = 9.808 \pm 0.040 \text{ m/s}^2$, which is consistent with a previously published value of g for our location.

Introduction

According to Newton's Law of Gravitation, the magnitude of the force, F , between two masses, m_1 and m_2 , separated by distance, r , is given by:

$$F = G \frac{m_1 m_2}{r^2} , \quad (1)$$

where G is the universal gravitational constant. An object in free-fall, near the surface of the Earth, will accelerate according to Newton's 2nd Law of Motion,

$$F = mg , \quad (2)$$

where g is the local acceleration due to gravity. Combining Equations 1 and 2 yields a value of g , given by

$$g = G \frac{M_e}{R_e^2} , \quad (3)$$

where M_e and R_e are the mass and radius of the Earth, respectively. According to the National Institute of Standards and Technology (NIST) the *standard acceleration due to gravity*, is 9.80665 m/s^2 .¹

Hidden in Equation 3 are a number of oversimplifying assumptions. A more careful analysis would need to take into account the fact that the Earth is a *non-inertial reference frame*: the rotation of the Earth on its axis introduces a Coriolis force, which reduces the acceleration due to gravity near the equator by about 0.5%, compared to the value of g at the poles. In addition, local variations in altitude, and the details of geological formations, could result in

differences in g around the globe, even at constant latitude. This latter effect is more subtle than the effect due to the Earth's rotation, but still measurable with sufficiently sensitive equipment.

The purpose of this experiment is to determine the local acceleration due to gravity, g . Our approach is to measure the time required for an object to fall through a measured distance, under the influence of the force of gravity. The general kinematic equation for uniform acceleration in one dimension is

$$y = y_o + v_o t + \frac{1}{2} a t^2 \quad , \quad (4)$$

where y_o is the initial position, v_o is the initial velocity, a is the acceleration and t is time. Since we are free to choose a convenient coordinate system, we will define the initial position to be $y_o = 0$ at $t = 0$, and positive direction to be downward. Our falling object will be released from rest, so we have $v_o = 0$. Thus, Equation (4) may be simplified to

$$y = \frac{1}{2} g t^2 \quad . \quad (5)$$

A plot of the measured distance of fall, y , as a function of the square of the measured time, t , should yield a straight line, whose slope is $\frac{1}{2} g$.

Apparatus and Procedure

A diagram of our free-fall apparatus is shown in Figure 1. The falling object is a steel ball-bearing (3/4" diameter, 27.880 ± 0.005 gram mass). We use a PASCO Model ME-9215A digital photogate timer, with Model ME-9207A Free-Fall Adapter to measure the free-fall time. The Free-Fall Adapter consists of two electrical contacts, which start and stop the timer. The ball-bearing is initially held in place in one of the contacts: a spring-loaded clamp, which triggers the timer when the ball is released. The second electrical contact is a strike pad, positioned directly below the ball, which stops the timer when the ball falls on it. The free-fall height is measured with a meter stick (millimeter divisions), from the bottom of the ball (clamped in the first contact) to the top of the strike pad (in its closed position). The resolution of the timer display is set to 0.1 ms.

The PASCO Model ME-9215A timers, as received from the manufacturer, are not always well-calibrated. Prior to making free-fall measurements, we perform a calibration procedure, so that we may be reasonably certain that we are reporting correctly measured times. The calibration procedure is described in detail in the Appendix.

For each of several different measured free-fall distances, we make 10 repeated measurements of the free-fall time. The distance is plotted against the square of the mean free-fall time, according to Eq. 5.

Results and Discussion

According to Eq. 5, a plot of the free-fall height versus the square of the time should yield a straight line through the origin, with a slope equal to $\frac{1}{2} g$. Figure 2 is a plot of our free-fall data. The times have been adjusted according to the timer calibration equation, as described in the Appendix. A linear least-squares fit to this set of data yields a reduced χ^2 of 0.31, which indicates a good fit. The y-intercept is -0.0016 ± 0.0049 m, which is consistent with zero. The slope of this plot is 4.904 ± 0.020 m/s².

Our experimental value for the acceleration due to gravity, g , derived from the slope of the plot in Fig. 2., is shown in Table I. This value is in good agreement with a previously published value of $g = 9.80118$ m/s² for our specific location (Pittsburgh, PA).²

Experimental result	Published Value	Difference over uncertainty
9.808 ± 0.040 m/s ²	9.80118 m/s ²	0.17

Table I. Acceleration Due to Gravity in Pittsburgh, PA
The difference between our experimental result and the published value, divided by the uncertainty on the difference, is less than 1, which indicates good agreement.

Conclusions

We have measured the local acceleration due to gravity with a precision of better than one half of one percent. However, this is not sufficiently precise to discern such subtle effects on g as the rotation of the Earth and altitude above sea level. Although the value of g is measurably different at various places around the world, the range of published values of g (from 9.782 m/s² near the equator to 9.825 m/s² near the pole m/s²)² is entirely encompassed by our margin of uncertainty. Nevertheless, our result of $g = 9.808 \pm 0.040$ m/s² is in good agreement with the published value for Pittsburgh, PA, and with the *standard acceleration due to gravity*.

References

1. http://physics.nist.gov/cgi-bin/cuu/Value?gn|search_for=gravity
2. Hugh D. Young, *University Physics*, 8th Ed. (Addison Wesley, 1992) p. 336.

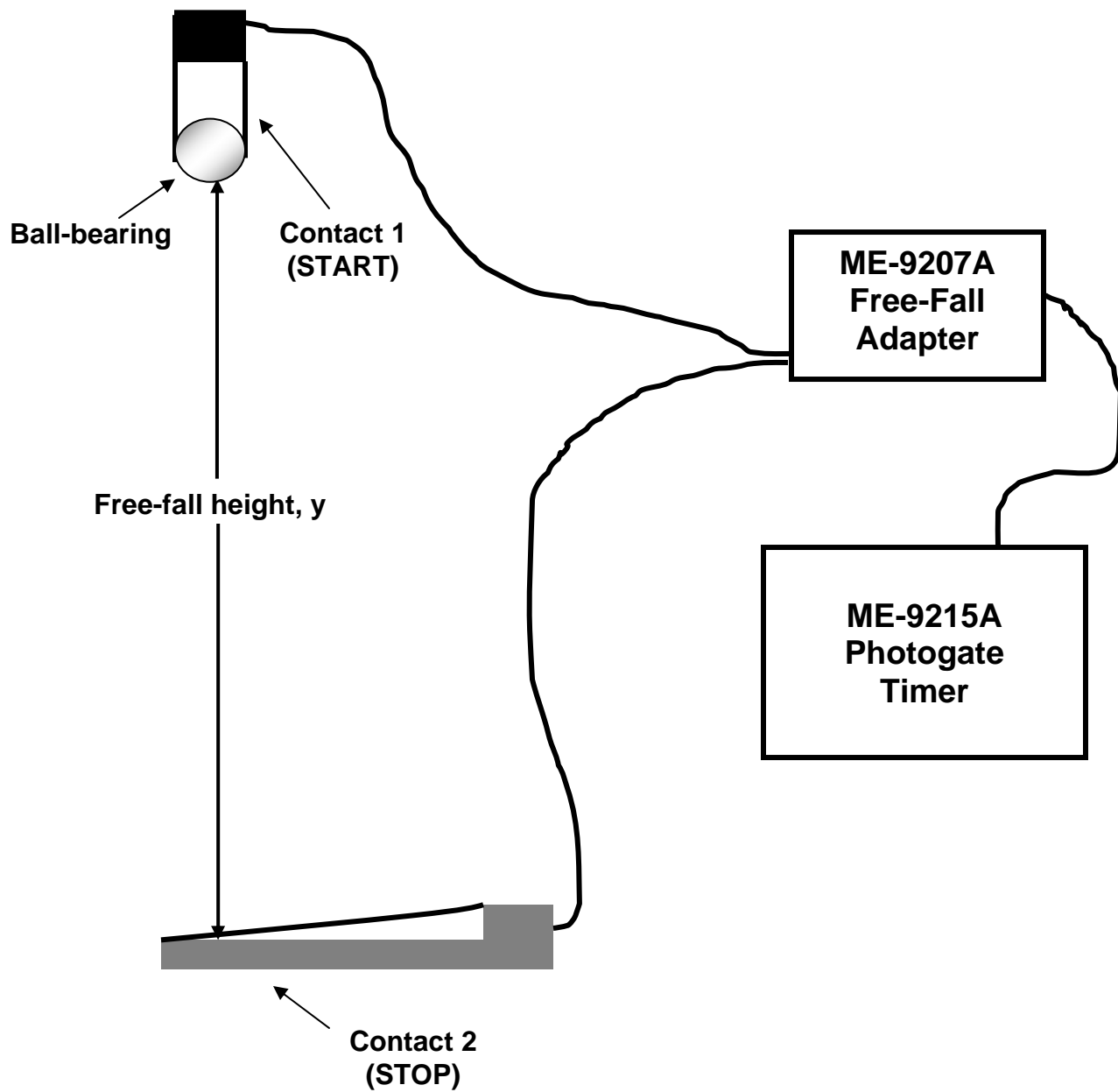


Figure 1. Free-Fall Apparatus

Steel ball-bearing is clamped in Contact 1. Free-fall height, y , is measured from bottom of ball to top of Contact 2 (in closed position). Timer starts when ball is released. Timer stops when ball strikes and closes Contact 2

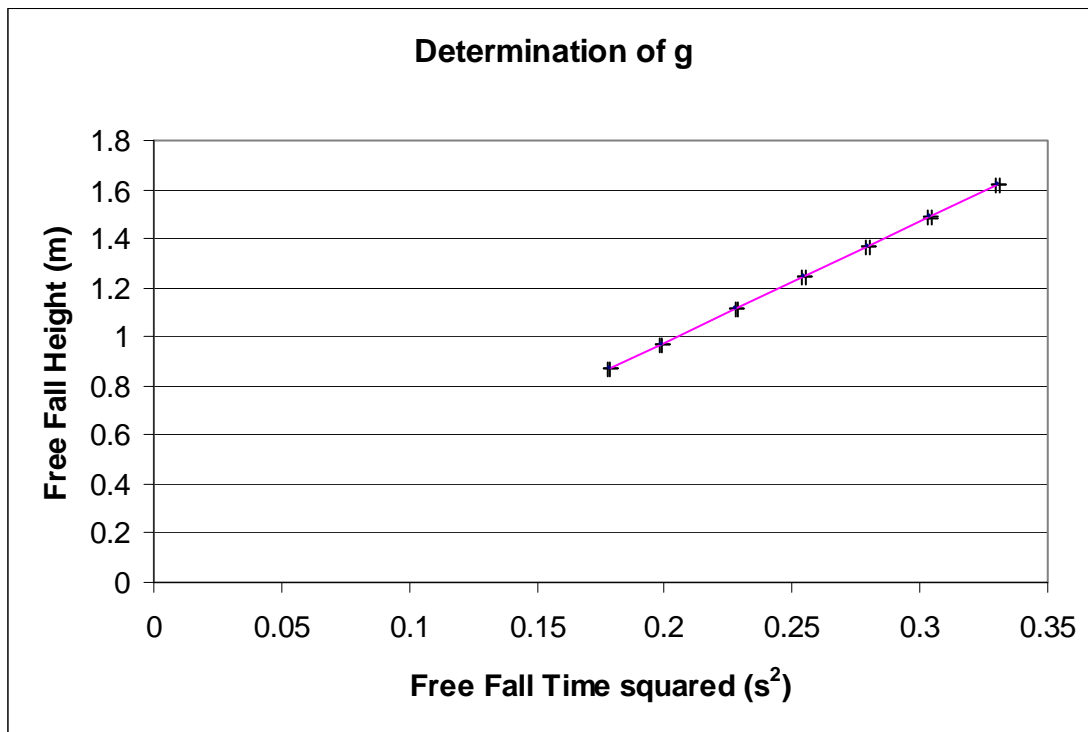


Figure 2.
Determination of local acceleration due to gravity, g , from free-fall data.

APPENDIX

Calibration of the PASCO ME-9215A Photogate Timer

The PASCO Model ME-9215A Photogate Timers are not always well-calibrated, as received from the manufacturer. Possible problems with the timer circuitry include a triggering delay, which may be different for the *start* pulse, compared to the *stop* pulse, resulting in a constant offset in time, and a quartz oscillator which may be running too fast or too slow. We will assume that these hypothetical calibration problems can be corrected by a linear adjustment, given by

$$t_{\text{calibrated}} = a + bt_{\text{measured}} \quad , \quad (\text{A-1})$$

where the intercept, a , is a constant offset in time, and b is a calibration scale factor. If the timer is well-calibrated, we should find $a = 0$ and $b = 1$.

To test the calibration of our timer, we use a setup shown in Figure A-1. We drive a mechanical oscillator (PASCO SF-9324) with the output from a well-calibrated digital sine wave generator (Stanford Research Systems DS335). An opaque obstacle, attached to the mechanical oscillator, moves up and down, interrupting the infrared beam of the Photogate Timer, at a frequency, f , displayed on the DS335 function generator. By definition, the period of oscillation of the obstacle is the reciprocal of the frequency. Thus, the calibration-standard time is given by

$$t_{\text{calibrated}} = \frac{1}{f} \quad , \quad (\text{A-2})$$

We make 10 repeated measurements of the period of oscillation with the photogate timer, at each of eight different frequencies, from 1.25 Hz to 3.00 Hz, in increments of 0.25 Hz. Over this range, the uncertainty on the frequency is ± 0.000001 Hz. The mean of the 10 measurements with the photogate is t_{measured} .

Figure A-2 is a plot of the calibration-standard time versus the measured time. A least-squares fit to the data yields a good fit (reduced χ^2 of 0.34) with a slope $b = 1.13520 \pm 0.00088$ and an intercept of $a = -0.00018 \pm 0.00034$ s. Although the intercept is consistent with zero, the slope clearly indicates that the timer is running too slow: the measured times need to be multiplied by a scale factor, which is greater than 1. Thus, in the final analysis, all of our times, measured with this particular photogate, must be adjusted according to

$$t_{\text{calibrated}} = 1.1352 \cdot t_{\text{measured}} \quad , \quad (\text{A-3})$$

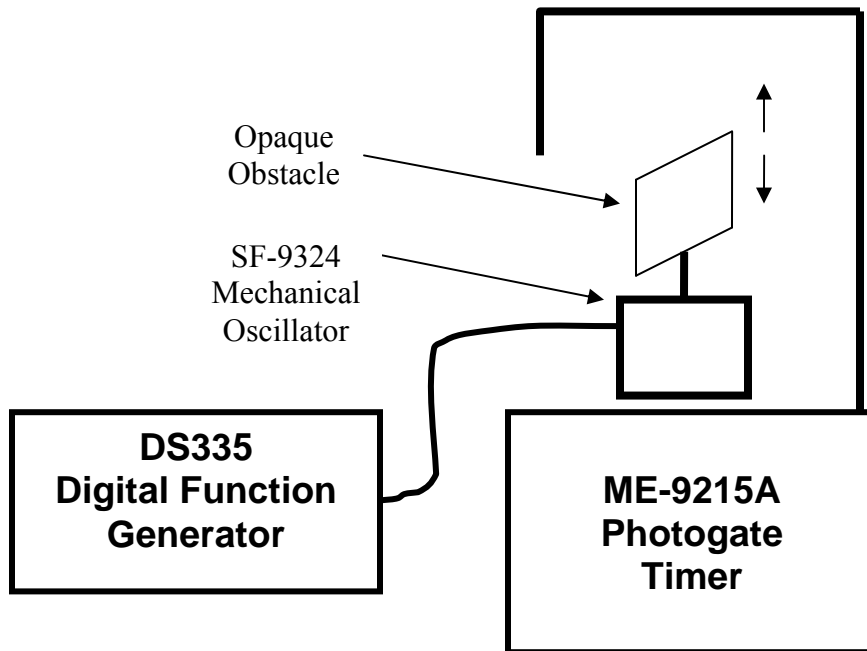


Figure A-1.
Timer calibration apparatus.

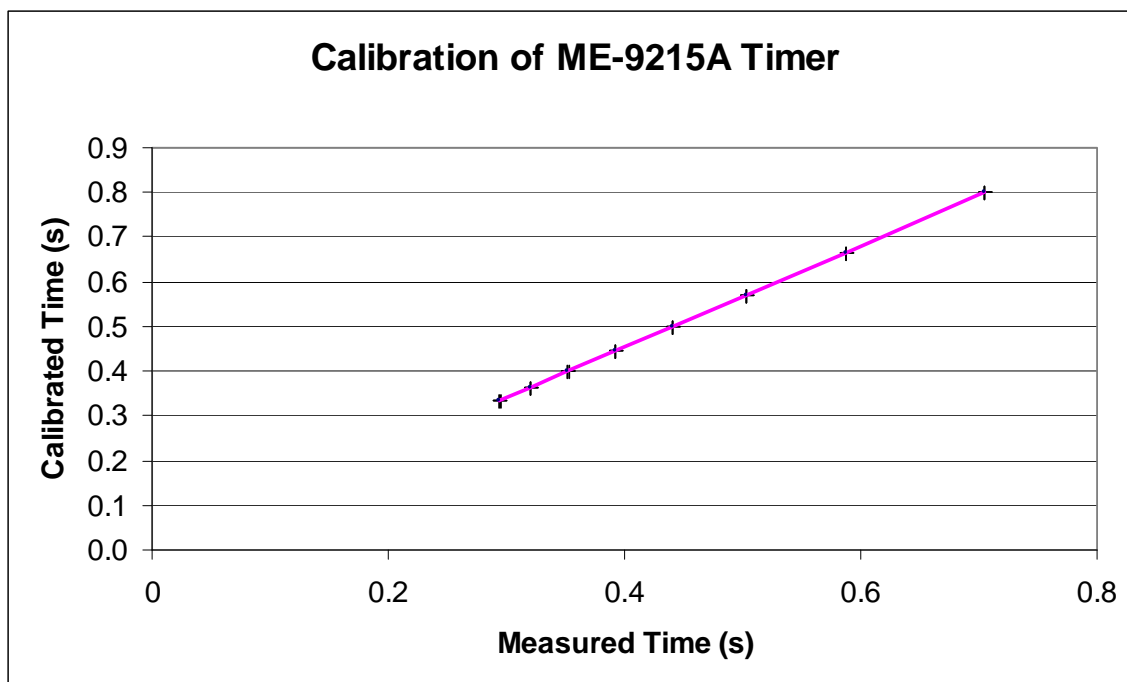


Figure A-2.
Calibration of ME-9215A Photogate Timer

4.3 Formal Report Checklist

The purpose of a formal report is to communicate the results of our work to an appropriate audience. Your goal is to persuade the audience that your experiment was well thought out and that your conclusions are supported by the results. The basic guidelines for doing this are summarized below. Your formal report must include ...

Abstract: A paragraph (or at most two) at the beginning of the report which conveys the essence and significance of your work. This is not an introduction, and there should be few details of procedure, if any. The abstract is intended to capture the attention of the audience and to persuade them that the rest of the report is worth their time.

Good organization: Divide the text of the report into numbered sections with heading as indicated below. The abstract is not numbered, but rather stands by itself after the title of the report. Important equations must be numbered in the margin. Each figure and table must be numbered in the same order in which you refer to it in the text of the report.

- I. **Introduction:** Describe the physics behind the experiment, explain what quantities must be measured and tell what information you will obtain from the data.
 - II. **Apparatus and Procedure:** Include just enough detail to explain how the measurements were made, but not a cookbook list of instructions. A carefully drawn schematic diagram of the apparatus (with figure number and descriptive caption) is an essential part of this section.
 - III. **Results and Discussion:** Explain how you made use of graphs and/or key equations to obtain the final results from the raw data. Every graph must have a figure number and a caption! Summarize your final results, including uncertainties and predicted values for comparison. Discuss the agreement or disagreement and any likely sources of systematic error which were not taken into consideration in the analysis. Results must have proper units and the correct number of significant figures.
 - IV. **Conclusions:** Summarize what you have learned about the behavior of the physical system. Make sure that your conclusions are supported by the results. This section gives the audience their final impression of your work.
-

Remember to proof-read your report to be sure that it includes everything you intended.